Georgia
Standards of Excellence
Curriculum Map

Mathematics

GSE Pre-Calculus

Georgia Department of Education

Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

Georgia Department of Education

GSE Pre-Calculus Curriculum Map							
1st Semester				**2nd Semester**			
Unit 1	**Unit 2**	**Unit 3**	**Unit 4**	**Unit 5**	**Unit 6**	**Unit 7**	**Unit 8**
Introduction to Trigonometric Functions *(4 – 5 weeks)*	**Trigonometric Functions** *(4 – 5 weeks)*	**Trigonometry of General Triangles** *(3 – 4 weeks)*	**Trigonometric Identities** *(3 – 4 weeks)*	**Matrices** *(3 – 4 weeks)*	**Conics** *(3 – 4 weeks)*	**Vectors** *(4 – 5 weeks)*	**Probability** *(4 – 5 weeks)*
MGSE9-12.F.IF.4 MGSE9-12.F.IF.7 MGSE9-12.F.IF.7e MGSE9-12.F.TF.1 MGSE9-12.F.TF.2 MGSE9-12.F.TF.5 MGSE9-12.F.TF.8	MGSE9-12.F.BF.4 MGSE9-12.F.BF.4d MGSE9-12.F.TF.3 MGSE9-12.F.TF.4 MGSE9-12.F.TF.6 MGSE9-12.F.TF.7	MGSE.9-12.G.SRT.9 MGSE.9-12.G.SRT.10 MGSE.9-12.G.SRT.11	MGSE9-12.F.TF.9	MGSE9-12.N.VM.6 MGSE9-12.N.VM.7 MGSE9-12.N.VM.8 MGSE9-12.N.VM.9 MGSE9-12.N.VM.10 MGSE9-12.N.VM.12 MGSE9-12.A.REI.8 MGSE9-12.A.REI.9	MGSE9-12.G.GPE.2 MGSE9-12.G.GPE.3 MGSE9-12.A.REI.7	MGSE9-12.N.CN.3 MGSE9-12.N.CN.4 MGSE9-12.N.CN.5 MGSE9-12.N.CN.6 MGSE9-12.N.VM.1 MGSE9-12.N.VM.2 MGSE9-12.N.VM.3 MGSE9-12.N.VM4 MGSE9-12.N.VM4a MGSE9-12.N.VM4b MGSE9-12.N.VM4c MGSE9-12.N.VM.5 MGSE9-12.N.VM.5a MGSE9-12.N.VM.5b MGSE9-12.N.VM.11	MGSE9-12.S.CP.8 MGSE9-12.S.CP.9 MGSE9-12.S.MD.1 MGSE9-12.S.MD.2 MGSE9-12.S.MD.3 MGSE9-12.S.MD.4 MGSE9-12.S.MD.5 MGSE9-12.S.MD.5a MGSE9-12.S.MD.5b MGSE9-12.S.MD.6 MGSE9-12.S.MD.7

These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units.
All units will include the Mathematical Practices and indicate skills to maintain.

NOTE: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics.

Grade 9-12 Key:

Number and Quantity Strand: RN = The Real Number System, Q = Quantities, CN = Complex Number System, VM = Vector and Matrix Quantities

Algebra Strand: SSE = Seeing Structure in Expressions, APR = Arithmetic with Polynomial and Rational Expressions, CED = Creating Equations,   REI = Reasoning with Equations and Inequalities

Functions Strand: IF = Interpreting Functions, LE = Linear and Exponential Models, BF = Building Functions, TF = Trigonometric Functions

Geometry Strand: CO = Congruence, SRT = Similarity, Right Triangles, and Trigonometry, C = Circles, GPE = Expressing Geometric Properties with Equations, GMD = Geometric Measurement and Dimension,
          MG = Modeling with Geometry

Statistics and Probability Strand: ID = Interpreting Categorical and Quantitative Data, IC = Making Inferences and Justifying Conclusions, CP = Conditional Probability and the Rules of Probability, MD = Using Probability
          to Make Decisions

Georgia Standards of Excellence Pre-Calculus
Curriculum Map Rationale

<u>**Unit 1:**</u> Students will use the unit circle to extend the domain of trigonometric functions to include all real numbers. Students will develop understanding of the radian measure of an angle, graph trigonometric functions, and derive and apply the Pythagorean identity.

<u>**Unit 2:**</u> Building on standards from Unit 1, students extend their study of the unit circle and trigonometric functions. Students will create inverses of trigonometric functions and use the inverse functions to solve trigonometric equations that arise in real-world problems.

<u>**Unit 3:**</u> Building on standards from Unit 1 and Unit 2, students will apply trigonometry to general triangles. Students will derive the trigonometric formula for the area of a triangle and prove and use the Laws of Sines and Cosines to solve problems.

<u>**Unit 4:**</u> Building on standards from the first three units, students will prove and use addition, subtraction, double, and half-angle formulas to solve problems.

<u>**Unit 5:**</u> Students will perform operations on matrices, use matrices in applications, and use matrices to represent and solve systems of equations.

<u>**Unit 6:**</u> Building on standards from previous courses, students will derive the equations of conic sections (parabolas, ellipses, and hyperbolas). Students will solve systems of a linear and quadratic equation in two variables.

<u>**Unit 7:**</u> Students will extend their understanding of complex numbers and their operations through graphical representations. Students will perform operations on vectors and use the operations to represent various quantities.

<u>**Unit 8:**</u> Students will extend their study of probability by computing and interpreting probabilities of compound events. Students will calculate expected values and use them to solve problems and make informed decisions.

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GSE Pre-Calculus Expanded Curriculum Map – 1st Semester			
Standards for Mathematical Practice			
1 Make sense of problems and persevere in solving them. **2** Reason abstractly and quantitatively. **3** Construct viable arguments and critique the reasoning of others. **4** Model with mathematics.		**5** Use appropriate tools strategically. **6** Attend to precision. **7** Look for and make use of structure. **8** Look for and express regularity in repeated reasoning.	
1st Semester			
Unit 1	**Unit 2**	**Unit 3**	**Unit 4**
Introduction to Trigonometric Functions	**Trigonometric Functions**	**Trigonometry of General Triangles**	**Trigonometric Identities**
Interpret functions that arise in applications in terms of the context **MGSE9-12.F.IF.4** Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. **Analyze functions using different representations** **MGSE9-12.F.IF.7** Graph functions expressed algebraically and show key features of the graph both by hand and by using technology. **MGSE9-12.F.IF.7e** Graph ~~exponential and logarithmic functions, showing intercepts and end behavior, and~~ trigonometric functions, showing period, midline, and amplitude. **Extend the domain of trigonometric functions using the unit circle** **MGSE9-12.F.TF.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **MGSE9-12.F.TF.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **Model periodic phenomena with trigonometric functions** **MGSE9-12.F.TF.5** Choose trigonometric functions to model periodic phenomena with	**Build new functions from existing functions** **MGSE9-12.F.BF.4** Find inverse functions. **MGSE9-12.F.BF.4d** Produce an invertible function from a non-invertible function by restricting the domain. **MGSE9-12.F.TF.3** Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π - x, π + x, and 2π - x in terms of their values for x, where x is any real number. **MGSE9-12.F.TF.4** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **MGSE9-12.F.TF.6** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **MGSE9-12.F.TF.7** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.	**Apply trigonometry to general triangles** **MGSE9-12.G.SRT.9** Derive the formula A = (1/2)ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **MGSE9-12.G.SRT.10** Prove the Laws of Sines and Cosines and use them to solve problems. **MGSE9-12.G.SRT.11** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	**Prove and apply trigonometric identities** **MGSE9-12.F.TF.9** Prove addition, subtraction, double and half-angle formulas for sine, cosine, and tangent and use them to solve problems.

specified amplitude, frequency, and midline. **Prove and apply trigonometric identities** MGSE9-12.F.TF.8 Prove the Pythagorean identity $(\sin A)^2 + (\cos A)^2 = 1$ and use it to find sin A, cos A, or tan A, given sin A, cos A, or tan A, and the quadrant of the angle.			

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GSE Pre-Calculus Expanded Curriculum Map – 2nd Semester

Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.	**5** Use appropriate tools strategically.
2 Reason abstractly and quantitatively.	**6** Attend to precision.
3 Construct viable arguments and critique the reasoning of others.	**7** Look for and make use of structure.
4 Model with mathematics.	**8** Look for and express regularity in repeated reasoning.

2nd Semester

Unit 5 **Matrices**	Unit 6 **Conics**	Unit 7 **Vectors**	Unit 8 **Probability**
Perform operations on matrices and use matrices in applications **MGSE9-12.N.VM.6** Use matrices to represent and manipulate data, e.g., transformations of vectors. **MGSE9-12.N.VM.7** Multiply matrices by scalars to produce new matrices. **MGSE9-12.N.VM.8** Add, subtract, and multiply matrices of appropriate dimensions. **MGSE9-12.N.VM.9** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. **MGSE9-12.N.VM.10** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. **MGSE9-12.N.VM.12** Work with 2 X 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. **Solve systems of equations** **MGSE9-12.A.REI.8** Represent a system of linear equations as a single matrix equation in a vector variable **MGSE9-12.A.REI.9** Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).	**Translate between the geometric description and the equation for a conic section** **MGSE9-12.G.GPE.2** Derive the equation of a parabola given a focus and directrix. **MGSE9-12.G.GPE.3** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. **Solve systems of equations** **MGSE9-12.A.REI.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*	**Use properties of rational and irrational numbers.** **MGSE9-12.N.CN.3** Find the conjugate of a complex number; use the conjugate to find the absolute value (modulus) and quotient of complex numbers. **Represent complex numbers and their operations on the complex plane** **MGSE9-12.N.CN.4** Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. **MGSE9-12.N.CN.5** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument 120°.* **MGSE9-12.N.CN.6** Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. **Represent and model with vector quantities** **MGSE9-12.N.VM.1** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, $|v|$, $\|v\|$, v). **MGSE9-12.N.VM.2** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.	**Use the rules of probability to compute probabilities of compound events in a uniform probability model** **MGSE9-12.S.CP.8** Apply the general Multiplication Rule in a uniform probability model, P(A and B) = [P(A)]x[P(B|A)] =[P(B)]x[P(A|B)], and interpret the answer in terms of the model. **MGSE9-12.S.CP.9** Use permutations and combinations to compute probabilities of compound events and solve problems. **Calculate expected values and use them to solve problems** **MGSE9-12.S.MD.1** Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. **MGSE9-12.S.MD.2** Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. **MGSE9-12.S.MD.3** Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* **MGSE9-12.S.MD.4** Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.

MGSE9-12.N.VM.3 Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors

MGSE9-12.N.VM.4 Add and subtract vectors.

MGSE9-12.N.VM4a Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

MGSE9-12.N.VM4b Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

MGSE9-12.N.VM4c Understand vector subtraction $v - w$ as $v + (-w)$, where $(-w)$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

MGSE9-12.N.VM.5 Multiply a vector by a scalar.

MGSE9-12.N.VM.5a Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

MGSE9-12.N.VM.5b Compute the magnitude of a scalar multiple cv using $\|cv\| = |c|v$. Compute the direction of cv knowing that when $|c|v = 0$, the direction of cv is either along v (for c > 0) or against v (for c < 0).

MGSE9-12.N.VM.11 Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?

Use probability to evaluate outcomes of decisions

MGSE9-12.S.MD.5 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

MGSE9-12.S.MD.5a Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*

MGSE9-12.S.MD.5b Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

MGSE9-12.S.MD.6 Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

MGSE9-12.S.MD.7 Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).