FOCUS ISSUE: the value of "Why?"

# Number Talks Build Numerical Reasoning

Strengthen accuracy, efficiency, and flexibility with these mental math and computation strategies.

By Sherry D. Parrish

Mary, a third grader, solves twelve minus five on her paper by crossing out the twelve and recording a zero above the ten and a twelve above the two. When asked to share why she solved the problem this way, Mary quickly replies, "Because you have to do it that way when the bottom number is bigger than the top number."

We would like to believe that this is a unique situation; however, our classrooms are filled with students like Mary who view mathematics as a collection of rules and procedures to memorize instead of a system of relationships to investigate and understand (NRC 2001).

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The math Process Standards highlighted in *Principles and Standards for School Mathematics* (NCTM 2000) and the National Research Council's Strands of Mathematical Proficiency discussed in *Adding It Up* have encouraged mathematics instruction to move beyond rote procedural knowledge, but these instructional shifts have yet to be consistently embraced or reflected in student performance nationally or internationally (NRC 2001). The recently drafted Common Core State Standards (CCSS) continue to build on these processes and proficiencies with eight Mathematical Practices and calls for instruction grounded in conceptual understanding and mathematical reasoning (CCSSI 2010).

How can educators make shifts in their instructional practices that foster sense making in mathematics and move forward in developing mathematical dispositions as outlined in each of these documents? *Classroom number talks*, five- to fifteen-minute conversations around purposefully crafted computation problems, are a productive tool that can be incorporated into classroom instruction to combine the essential processes and habits of mind of doing math. During number talks, students are asked to communicate their thinking when presenting and justifying solutions to problems they solve mentally. These exchanges lead to the development of more accurate, efficient, and flexible strategies. What does it mean to compute accurately, efficiently, and flexibly? *Accuracy* denotes the ability to produce an accurate answer; *efficiency* denotes the ability to choose an appropriate, expedient strategy for a specific computation problem; and *flexibility* refers to the ability to use number relationships with ease in computation (Russell 2000). Take the problem 49 × 5 as an example. A student exhibits these characteristics if she can use the relationship between 49 and 50 to think about the problem as 50 × 5 and then subtract one group of 5 to arrive at the answer of 245. A more detailed example of how to develop these characteristics follows.

## A classroom number talk

As I walk into Ms. Johnson's fourth-grade classroom, the class is gathered on the floor discussing solutions for 16 × 25, a problem the students are solving mentally during their classroom number talk. Three student solutions are posted on the board for consideration: 230, 400, and 175. The soft hum of conversation can be heard as partners discuss their strategies and share their thinking. Johnson reminds the partners to discuss whether all answers are reasonable by quietly pointing to the class motto posted around the room: "Does it make sense?" She eavesdrops on their conversations until she has an understanding of their ideas and then brings students back for a whole-group number talk.

**Please finish your last thought with your partner. Who would be willing to share their reasoning with the class?**
[*Madeline*] Steven and I disagree with 230 and 175 because the answer has to be over 250.

**Why do you think the solution should be more than 250?**
[*Steven*] Because 10 × 25 is 250, and you still have more to multiply. So we can rule out 230 and 175.
[*Most of the students reply.*] Agree!
[*Marquez*] My partner and I can prove that 400 is the right answer.

**How did you solve 25 × 16?**
[*Marquez*] We knew that 4 × 25 was 100 and that there were four groups of 4 × 25 in 16 × 25. That would be the same as four 100s, which would equal 400.

**Is this** [*capturing Marquez's strategy on the white board*, 16 × 25 = 4 × (4 × 25)] **how you thought about the problem?**
[*Marquez*] Yes. I knew I could break 16 into its factors of 4 × 4 and then I could multiply it in any order, but I used 4 × 25 first because that made a quick 100.
[*Anastasia*] I got 230, but I don't see why my answer isn't right. I multiplied each part.

**Why don't you tell us how you thought about the problem.**
[*Anastasia*] I multiplied 20 × 10 and that was 200, then I multiplied 5 × 6 and got 30. When I added those answers together, I got 230.

[*Johnson scribes Anastasia's strategy on the board for the students to consider.*]
[*Jack*] I don't think you multiplied all the parts of the numbers; you missed some groups.

**Let's use an open array to help us think about Anastasia's way and how she broke apart the numbers.** [*She draws an array representing 16 × 25 on the board.*] **"Anastasia, where did your 20 × 10 come from?"**
[*Anastasia*] The 20 came from the 25, and the 10 came from the 16. I broke the 25 into a 20 and a 5 and the 16 into a 10 and a 6 and then multiplied 20 × 10 and 6 × 5.

[*Johnson scribes Anastasia's strategy on the open array, recording the multiplication expressions in the appropriate spaces in the array* (*see* **fig. 1**).]
[*Nicole*] I get it! She didn't use all the groups!
[*Anastasia*] Oh, I see! I left out the 10 groups of 5 and 6 groups of 20. If I add the 50 from 10 × 5 and the 120 from 6 × 20 plus my 230, I would get 400.
[*Jack*] Amanda and I also got 400, and we multiplied every part, too. We did 6 × 5, 6 × 20, 10 × 5,

FIGURE 1

Anastasia used an open array as a strategy to solve the multiplication problem.

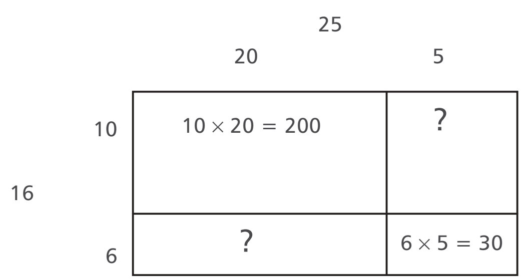
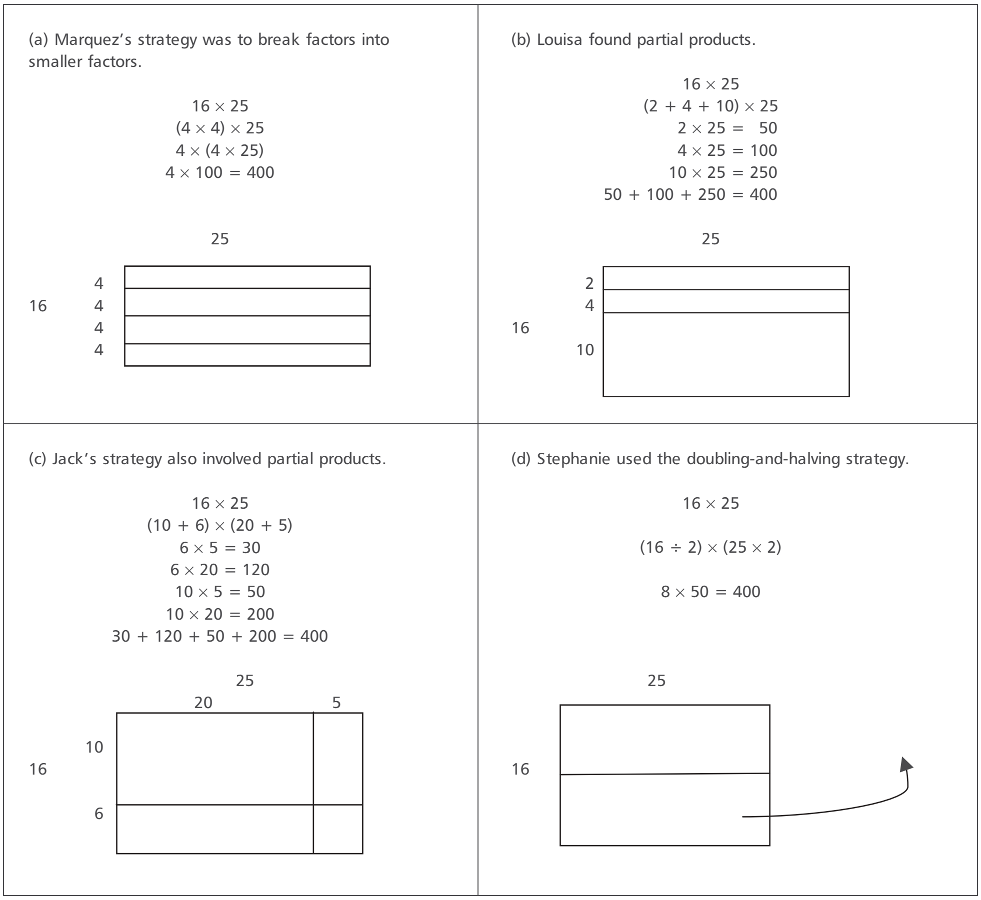


FIGURE 2

The class tried several student strategies for 16 × 25 to see if they would work for any multiplication problem.




and 10 × 20; then we added all these answers to get 400.

[*Paulette*] Looking at the open array, I see why this way works. But it seems hard to keep track of all of the numbers unless you draw that.

[*Louisa*] Blakely and I also got 400, but we only broke apart the 16. We broke the 16 into 4, 2, and 10 and then multiplied each part by 25: 4 × 25 is 100, 2 × 25 is 50, and 10 × 25 is 250. All the parts added up to 400.

[*Recording Louisa's strategy*] **How do you know if you still have sixteen 25s?**

[*Louisa*] Because 4 + 2 + 10 is 16. We just broke 16 apart into easy numbers for us to multiply. We can use an open array to show why it works. [*As Louisa draws the corresponding array, Jack agrees that this is a more efficient way to multiply* 16 × 25.]

[*Stephanie*] Michael and I doubled and halved. We doubled 25 to get 50, and then we halved 16 to get 8. All we had to do was multiply 8 times 50 to get 400. We know this works, because one side of the array doubles as the other side halves—the space inside the array stays the same.

[*Several students exclaim at once*] That's *really* fast!

**It looks like everyone is agreeing with 400, and we're beginning to think about efficiency as well as accuracy** [*recording Stephanie's strategy*]**. I'd like you to find a place to work with your partners to test each of these ideas** [*see* **fig. 2**] **to see if they would work for any multiplication problem.**

## A number talk's key components

We can extract five essential components of a classroom number talk from Johnson's classroom vignette: the classroom environment and community, classroom discussions, the teacher's role, the role of mental math, and purposeful computation problems (Parrish 2010).

### 1. Classroom environment and community

Building a cohesive classroom community is essential for creating a safe, risk-free environment for effective number talks. Students should be comfortable in offering responses for discussion, questioning themselves and their peers, and investigating new strategies. The culture of the classroom should be one of acceptance based on a common quest for learning and understanding. It takes time to establish a community of learners built on mutual respect, but if you consistently set this expectation from the beginning, students will respond.

A first step toward establishing a respectful classroom learning community is acceptance of all ideas and answers—regardless of any obvious errors. Rich mathematical discussions cannot occur if this expectation is not in place. We must remember that wrong answers are often rooted in misconceptions, and unless these ideas are allowed to be brought to the forefront, we cannot help students confront their thinking. Students who are in a safe learning environment are willing to risk sharing an incorrect answer with their peers to grow mathematically.

Expecting acceptance of all ideas without evaluative comments is important. Educators can model this trait by recording all answers to be considered without giving any verbal or physical expressions that indicate agreement or disagreement with any answer. Teachers may need to practice having a "blank face." Students look to teachers as the source of correct answers. Part of building a safe learning community is to shift this source of knowledge to the students by equipping them to defend the thinking behind their solutions.

### 2. Classroom discussions

A successful number talk is rooted in communication. During a number talk, the teacher writes a problem on the board and gives students time to mentally solve it. Students start with their fists held to their chests and indicate when they are ready with a solution by quietly raising a thumb. Once students have found an answer, they are encouraged to continue finding efficient strategies while others are thinking. They

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indicate that they have found other approaches by raising another finger for each solution. This quiet form of acknowledgment allows time for students to think, while the process continues to challenge those who already have an answer. When most of the students have indicated they have a solution and a strategy, the teacher calls for answers. All answers—correct and incorrect—are recorded on the board for students to consider.

The benefits of sharing and discussing computation strategies are highlighted below. Students have the opportunity to do the following:

- **Clarify** thinking.
- **Investigate and apply** mathematical relationships.
- **Build** a repertoire of efficient strategies.
- **Make** decisions about choosing efficient strategies for specific problems.
- **Consider and test** other strategies to see if they are mathematically logical.

Number talks may naturally lead to student investigations of strategies, as we saw at the conclusion of Johnson's number talk. Students can test numerous problems using strategies that surface during the number talk. By keeping a record of which strategies work and under what parameters, they can share their findings with the class and make a group decision on whether the strategy is mathematically logical, able to be generalized, and can be applied in all situations. This is another way to transfer the ownership of learning to students. We can see how this might unfold in the classroom by continuing to follow the conversations with Johnson's students after they have investigated strategies exhibited during their class number talk.

**I noticed many of you chose to investigate the doubling-and-halving strategy that Stephanie and Michael used in our number talk. Would someone share what you discovered?**
[*Jake*] Marquez and I first tried doubling and halving by testing smaller numbers like 6 × 9, 8 × 14, and 5 × 15; it worked with the first two problems but not with 5 × 15.

**Why do you think it didn't work with 5 × 15?**
[*Marquez*] We think it's because 5 and 15 are odd numbers, and you can't halve an odd number.

> "The heart of number talks is **classroom conversations** focused on making sense of mathematics."

[*Blakely*] Yes, you can. If you halve 5, you get 2 and a half, and half of 15 is 7 and a half.
[*Anastasia*] I agree with Blakely; you could do it, but it wouldn't necessarily make it an easier way to solve the problem.
[*Desmond*] Nicole and I also tried doubling and halving with two odd numbers, 9 × 13, but we ended up with fractions, and that made the problem harder to solve.

**If doubling and halving isn't necessarily an efficient strategy when multiplying two odd numbers, when is it an efficient method?**
[*Marilyn*] Josh and I tested problems when both numbers were even and also when one number was even and one was odd, but it seemed to make the problems easier to solve when one factor was even and the other was odd.

**Can you give us an example?**
[*Marilyn*] First we tried just even numbers with 12 × 16, and then doubled and halved to change the problem to 6 × 32, then 3 × 64. This made it a one-digit times a two-digit number, but it still wasn't a simple problem. We even tried doubling the 12 factor and halving the 16, but we ended up with 96 × 2. Next, we tried an odd number times an even number, and it seemed easier. We did 12 × 15 and doubled and halved to get 6 × 30, then 3 × 60; it was fast and efficient.
[*Roberto*] Juan and I found the same thing—it seems like a more efficient way when you have an odd number multiplied with an even number.

**I'm seeing lots of nods of agreement with this statement. I'd like us to keep investigating to see if our generalization holds true and if there are any exceptions to our rule.**

### 3. The teacher's role

As educators, we are accustomed to assuming the roles of telling and explaining. Teaching by telling is the method many of us experienced

> Simply defined, number talks are five- to fifteen-minute classroom conversations around **purposefully crafted computation** problems that are solved mentally.

as students, and we may have a tendency to emulate this model in our own practice. Because a primary goal of number talks is to help students make sense of mathematics by building on mathematical relationships, our role must shift from being the sole authority in imparting information and confirming correct answers to assuming the interrelated roles of facilitator, questioner, listener, and learner.

Since the heart of number talks is classroom conversations focused on making sense of mathematics, it is appropriate for the teacher to move into the role of facilitator. Keeping the discussion focused on the important mathematics and helping students learn to structure their comments and wonderings during a number talk is essential to ensuring that the conversation flows in a natural, meaningful manner. As a facilitator, you must guide students to ponder and discuss examples that build on your purposes. By posing such questions as, How does Joey's strategy connect to the ideas in Renee's strategy? you lead conversations to build on meaningful mathematics.

As we move toward listening to our students' thinking instead of concentrating on only a final, correct answer and one procedure, we will begin to ask open-ended questions. By changing our question from, What answer did you get? to How did you solve this problem? we will be able to understand how students are making sense of the mathematics.

### 4. Role of mental math

If students' math experiences have primarily focused on learning and practicing the standard U.S. algorithms for each operation, they may be resistant to looking at problems from other perspectives. Some students may try to visualize the problems vertically and will even write the problem with their fingers on the floor or in the air to remain consistent with the paper-and-pencil algorithms they may have learned. Mental computation is a key component of number talks because it encourages students to build on number relationships to solve problems instead of relying on memorized procedures. One purpose of a number talk is for students to focus on number relationships and use these relationships to develop efficient, flexible strategies with accuracy. When students approach problems without paper and pencil, they are encouraged to rely on what they know and understand about the numbers and how they are interrelated. Mental computation encourages them to be efficient with the numbers to avoid holding numerous quantities in their heads.

Mental computation also helps strengthen students' understanding of place value. By looking at numbers as whole quantities instead of discrete columns of digits, students must use their knowledge and understanding of place value. During initial number talks, problems are often written in horizontal format to encourage student's thinking in this realm. A problem such as 199 + 199 helps illustrate this reasoning. By writing this problem horizontally, you encourage a student to think about and use the value of the entire number. A student with a strong sense of number and place value should be able to consider that 199 is close to 200; therefore, 200 + 200 is 400 minus the two extra units for a final answer of 398.

Recording this same problem in a vertical format can encourage students to ignore the magnitude of each digit and its place value. A student who sees each column as a column of units would not be using real place values in the numbers if they are thinking about 9 + 9, 9 + 9, and 1 + 1.

### 5. Purposeful computation problems

Crafting problems that guide students to focus on mathematical relationships is an essential part of number talks that is used to build mathematical understanding and knowledge. The teacher's goals and purposes for the number talk should determine the numbers and operations that are chosen. Carefully planning before the number talk is necessary to design "just right" problems for students.

For example, if the goal is to help students develop multiplication strategies that build on using tens, starting with numbers multiplied by 10 followed by problems with 9 in the units column creates a situation where this type of strategy is important. Such problems as 20 × 4, 19 × 4, 30 × 3, 29 × 3, 40 × 6, and 39 × 6 lend themselves to strategies where students use tens as friendly, or landmark, numbers. In the problem 19 × 4, the goal would be for students to think about 20 × 4 and subtract 4 from the product of 80, because they added on one extra group of 4. Other problem sets can then be designed to elicit a similar approach. In later number talks, the teacher would begin with a number with 9 in the units column without starting with the multiple of 10 (see **fig. 3**).

**FIGURE 3**

Shown are examples of purposefully designed computation problems.

| Focus on using doubling and halving in multiplication | Focus on using landmark numbers in multiplication |
|---|---|
| 1 × 48 | 3 × 50 |
| 2 × 24 | 3 × 49 |
| 4 × 12 | 5 × 200 |
| 8 × 6 | 5 × 199 |
| 16 × 3 | |

Does this mean that given a well-crafted series of problems, students will always develop strategies that align with the teacher's purposes? No. Numerous strategies exist for any given problem; however, specific types of problems typically elicit certain strategies. Take, for instance, the same 19 × 4 problem that was crafted to target students' thinking using tens. Students could approach this problem in a variety of ways, including but not limited to the following:

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> "Give yourself license to be a **learner** along with your students."

- **Break the 19** into 10 + 9; then multiply 10 × 4 and 9 × 4 and combine these products.
- **Break the 4** into 2 + 2; then multiply 2 × 19 and 2 × 19 and combine these products.
- **Add** 19 + 19 + 19 + 19, or add 4 nineteen times.

However, a mixture of random problems, such as 39 × 5, 65 – 18, and 148 + 324, do not lend themselves to a common strategy. This series of problems may be used as practice for mental computation, but it does not initiate a common focus for a number talk discussion.

## Taking the first steps

Making the shift toward teaching for understanding by encouraging students to develop mental math strategies can often be overwhelming. Many of us are comfortable with telling students how to solve a problem but avoid focusing on student-invented strategies because doing so may feel foreign and intimidating. As you begin implementing number talks, consider starting first with smaller numbers, such as basic facts. Using basic facts as a starting place is an excellent way to establish that many ways exist to view and approach a problem. With the fact 6 + 7, we see multiple ways for students to think about this problem:

- **Use** doubles (6 + 6 = 12 plus one more; 7 + 7 = 14 minus one).
- **Make** a quick ten (6 can be split into 3 + 3 and 3 + 7 = 10 plus three more).
- **Count on** or count all (7, 8, 9, 10, 11, 12, 13).

Give yourself the license to be a learner along with your students and to question, "Does it make sense?" When students become accurate, efficient, and flexible with their mental math strategies, you can then transition to their regular paper-and-pencil computation practice. Encourage them to incorporate their mental math strategies from number talks with other algorithms by solving each problem in two different ways. This will not only help serve as a system of checks and balances for accuracy but also help students develop and maintain flexibility in thinking about numbers.

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