



Georgia's K-12 Mathematics Standards

[K-12 Mathematics Glossary](#)

[How to Use This Glossary](#)

[Get Started](#)

How to Use This Glossary

On the following pages, you will find mathematical terms and their definitions. In most cases, you will also find visuals and/or examples that have been added for clarification. Links to related mathematical terms within this glossary may also be listed after the definition. In addition, for some terms and definitions, links to outside resources may be provided as an opportunity to explore mathematical concepts more deeply.

In addition to the visuals and links, three features have been added to make this glossary more user friendly and limit the need to scroll.

1. [On the next page](#), you will see a colorful, interactive, alphabetical keyboard (see image below). Clicking a letter will take you to a page with all terms that begin with this letter. From there, you can click on the term you would like to explore.



2. At the top of each mathematical term page, there is a colored bar with the letter of the alphabetical section you are exploring (see sample image below). To go back to the list of terms beginning with this letter, click the colored bar.



3. At the bottom right of each page is a **Back to Start** button. Clicking this button will take you back to the interactive, alphabetical keyboard where you can choose another letter to explore.



Click a letter to explore mathematical terms associated with that letter.



A

[Absolute Value](#)

[Acute Angle](#)

[Addition and Subtraction within
5, 10, 20, 100, 1000 or 10000](#)

[Additive Comparison](#)

[Additive Identity Property of 0](#)

[Additive Inverses](#)

[Adjacent](#)

[Algorithm](#)

[Angle](#)

[Area Model](#)

[Assess Reasonableness](#)

[Assessment](#)

[Associative Property of Addition](#)

[Associative Property of Multiplication](#)

[Automaticity](#)

A

Absolute Value. The distance of a number from 0. Since absolute value measures distance, the value is always positive. The absolute value symbol is a pair of vertical lines: $| |$. For example, $|-4| = 4$ is read, “The absolute value of -4 equals 4.”

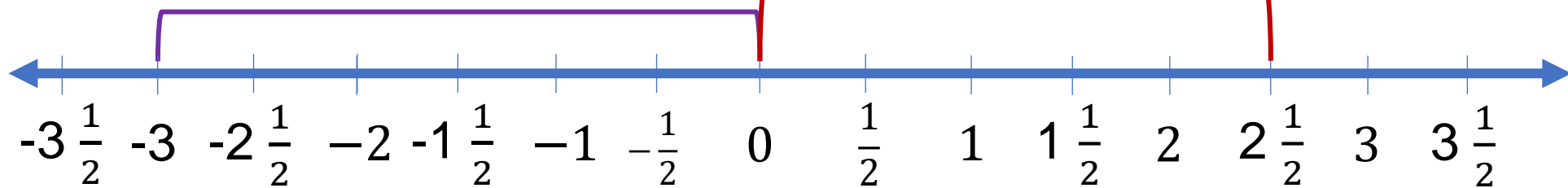
Examples:

The absolute value of -3 is 3
because -3 is three units
away from 0.

$$|-3| = 3$$

The absolute value of $2\frac{1}{2}$ is $2\frac{1}{2}$
because $2\frac{1}{2}$ is two and a half
units away from 0.

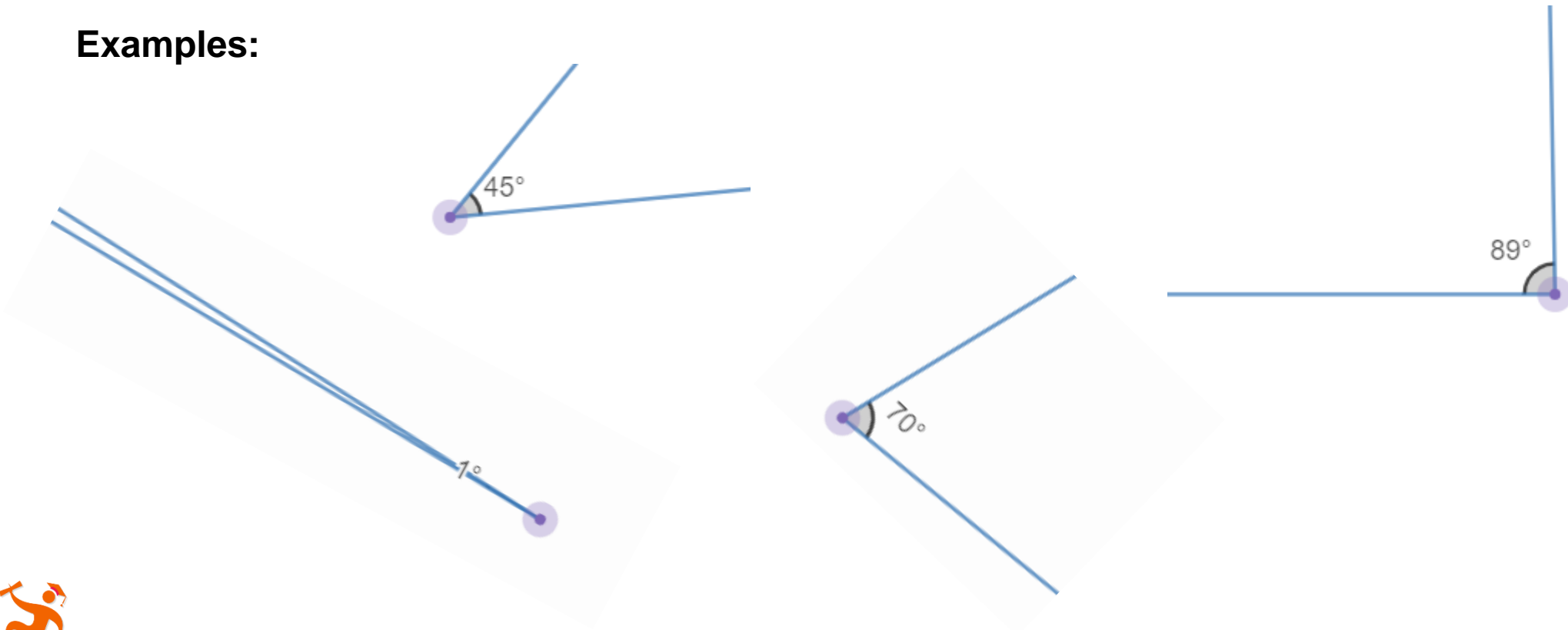
$$\left|2\frac{1}{2}\right| = 2\frac{1}{2}$$



A

Acute Angle. An angle whose measure is between 0° and 90° . Below are a few examples of acute angles. See also: [Angle](#), [Obtuse Angle](#), and [Right Angle](#).

Examples:



A

Addition and Subtraction within 5, 10, 20, 100, 1000, or 10000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, 0-100, 0-1000 or 0-10000, respectively.

Examples:

**Addition & Subtraction
within 10**

$$4 + 6 = \mathbf{10}$$

$$\mathbf{9} - 5 = 4$$

**Bold values must be
within the range 0-10**

**Addition & Subtraction
within 20**

$$14 + 5 = \mathbf{19}$$

$$\mathbf{16} - 5 = 11$$

**Bold values must be
within the range 0-20**

**Addition & Subtraction
within 100**

$$55 + 34 = \mathbf{89}$$

$$\mathbf{55} - 18 = 37$$

**Bold values must be
within the range 0-100**

**Addition & Subtraction
within 1000**

$$185 + 227 = \mathbf{412}$$

$$\mathbf{628} - 92 = 536$$

**Bold values must be
within the range 0-1000**

A

Additive Comparisons. These types of comparisons focus on the difference or distance between two quantities. The relationship between addition and subtraction can be naturally explored with additive comparisons. See also: [Additive Inverse](#), [Multiplicative Comparison](#)



$$\text{Whole} = \text{Part} + \text{Difference}$$

$$\text{Difference} = \text{Whole} - \text{Part}$$

Example:

Difference between 17 and 9



$$17 - 9 = 8$$

$$9 + 8 = 17$$

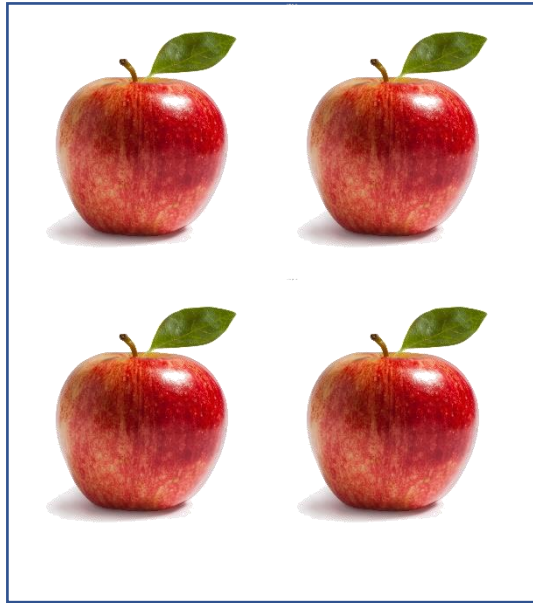
9 10 11 12 13 14 15 16 17

[Back to Start](#)

A

Additive Identity Property of 0. Also known as the identity property of addition, it states that adding 0 to any number results in the number itself. See *also*: [Table 3 in this Glossary](#).

Example:



4

and

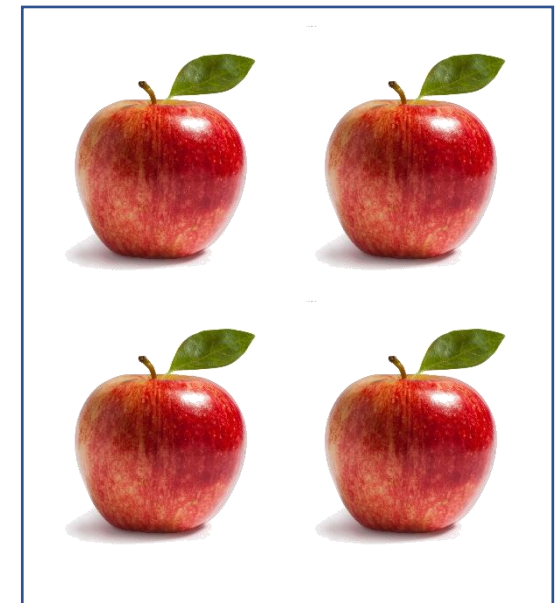
+



0

is

=



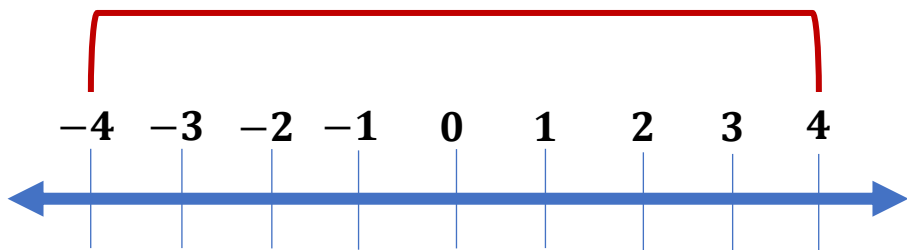
4

A

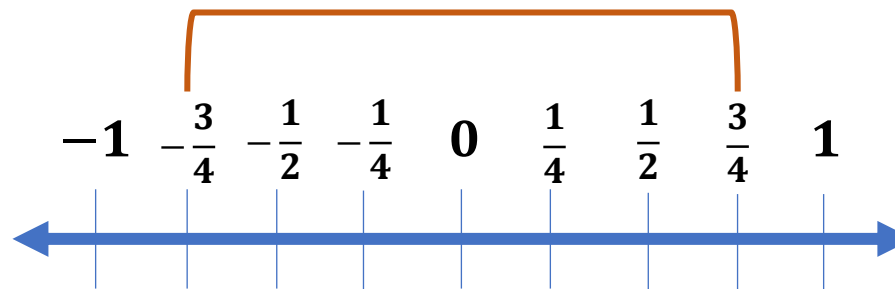
Additive Inverses. Two numbers whose sum is 0 are additive inverses of one another.

Additive inverses are equidistant from zero on a number line.

Example: 4 and -4 are additive inverses of one another because $4 + (-4) = (-4) + 4 = 0$.



Example: $\frac{3}{4}$ and $-\frac{3}{4}$ are additive inverses of one another because $\frac{3}{4} + (-\frac{3}{4}) = (-\frac{3}{4}) + \frac{3}{4} = 0$.

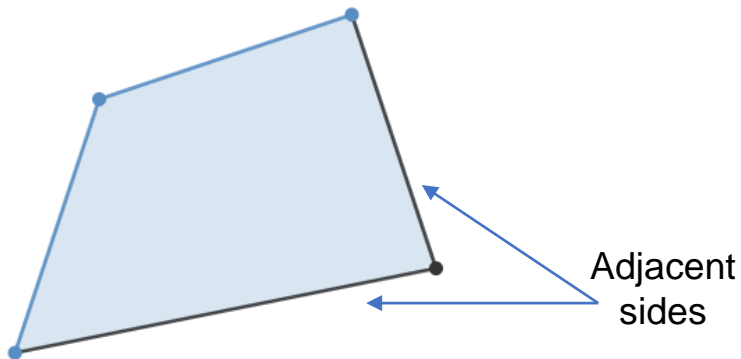


A

Adjacent. As in normal usage, the word 'adjacent' in geometry refers to items that are next to each other in a figure. The term 'adjacent' is usually applied to lines, arcs, or angles.

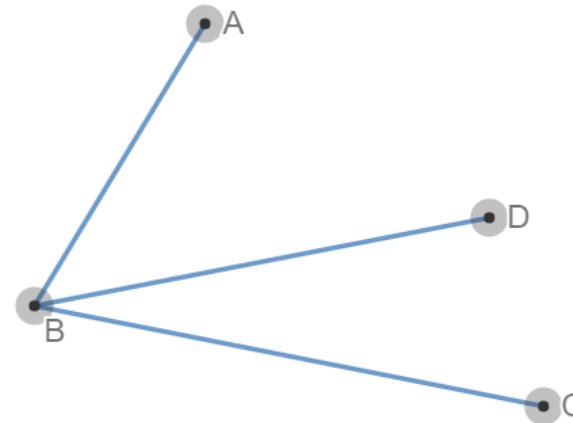
Examples:

Adjacent Sides



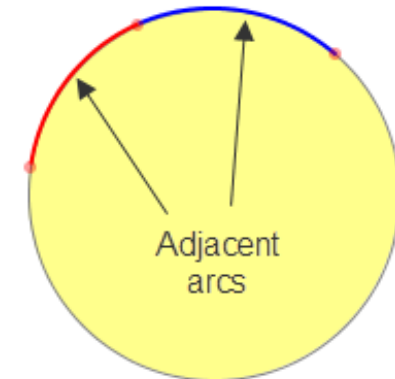
In a polygon, adjacent sides are next to each other and share a common vertex. The common vertex in the image above is black.

Adjacent Angles



Adjacent angles share a common side and vertex. Angles ABC and DBC are adjacent. These angles share side DB and vertex B.

Adjacent Arcs

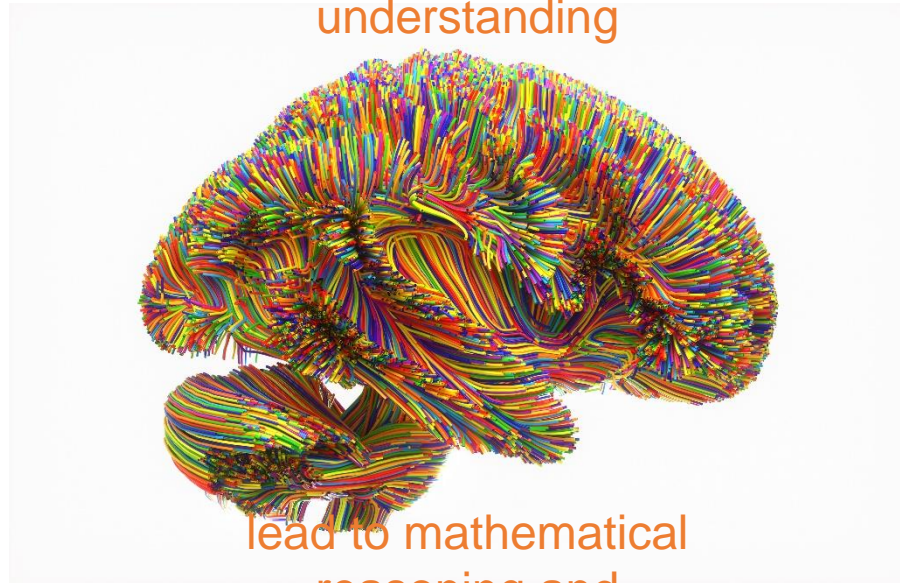


On the circumference of a circle, adjacent arcs are next to each other. They can be combined to make a single, larger arc.

A

Algorithm. A method or procedure for carrying out a calculation. See *also*: [Computation Algorithm](#), [Computation Strategy](#)

Algorithms based on
understanding

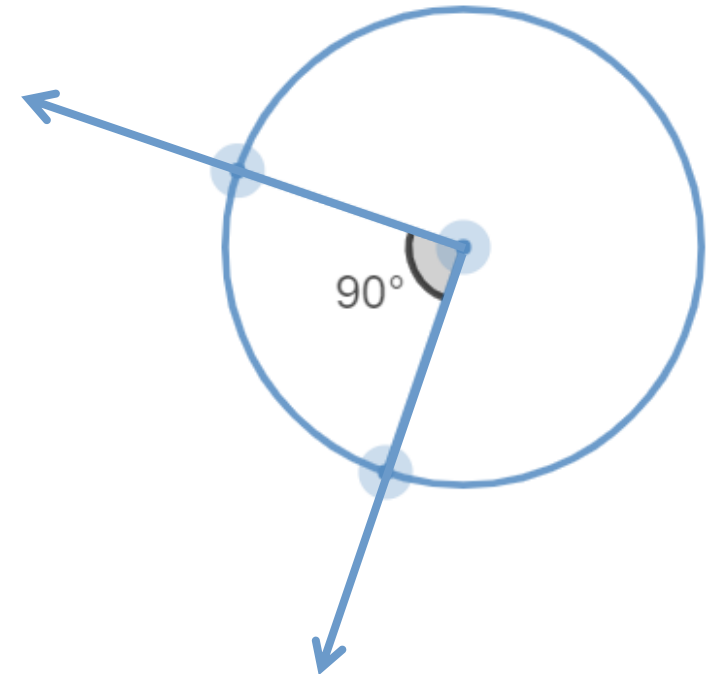
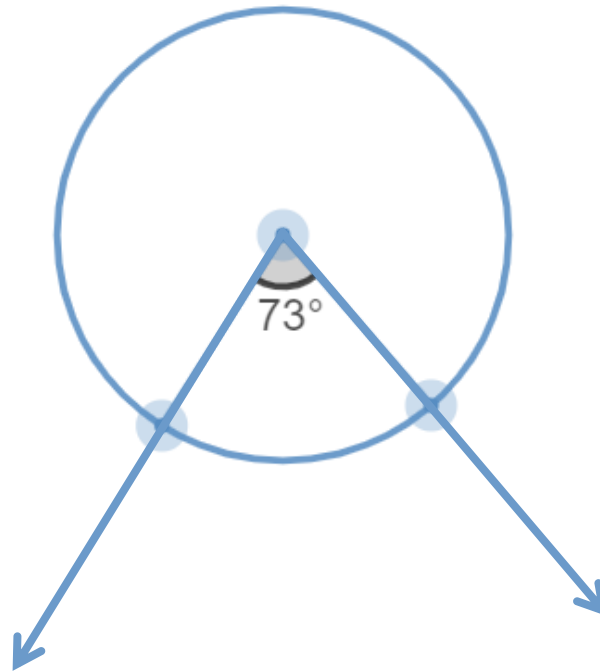
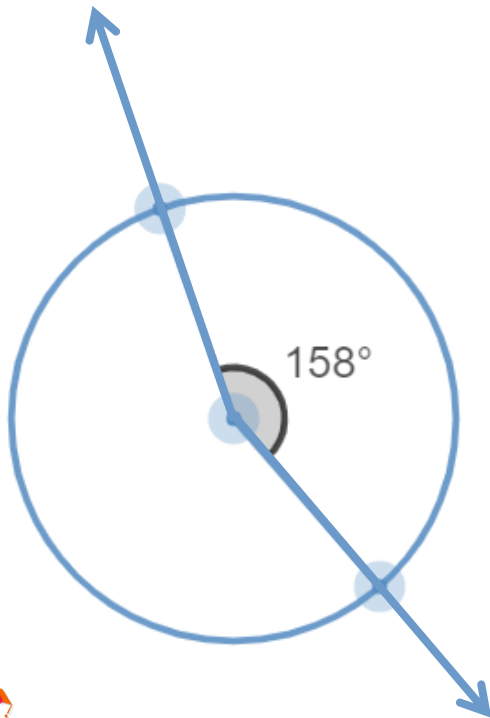


lead to mathematical
reasoning and
comprehension!

A

Angle. A shape formed by two rays sharing a common endpoint (vertex). Angles can be measured in degrees or radians. See also: [Acute Angle](#), [Obtuse Angle](#), [Right Angle](#), [Vertex](#)

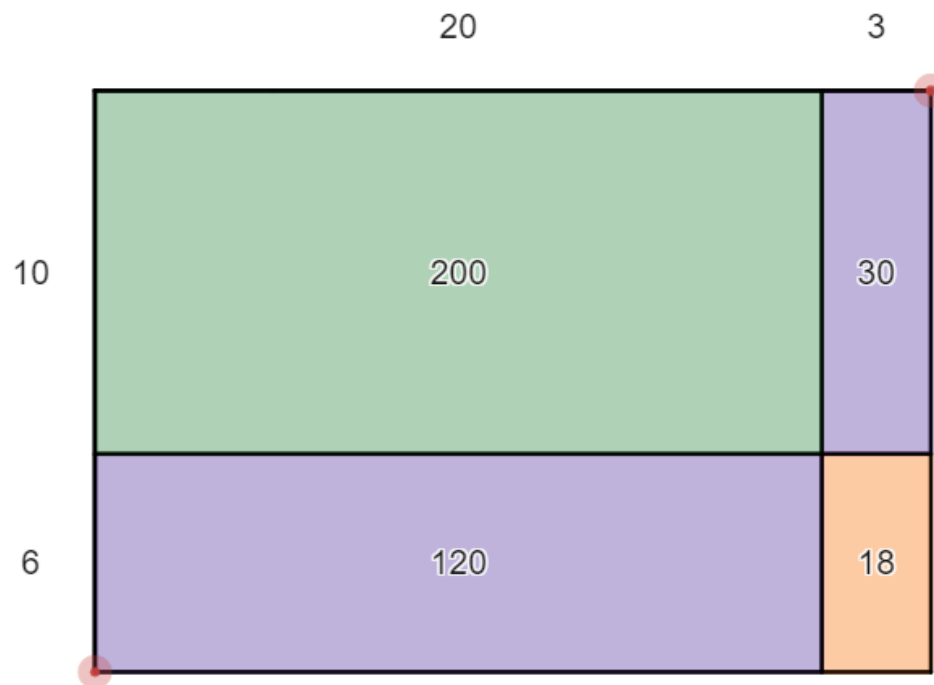
Examples:



A

Area Model. A model for multiplication and/or division problems, in which the length and width of a rectangle represents the factors, and the partial areas represent partial products. Area models can be used for division as well as multiplication.

Example: 23×16



Click the image to try an interactive area model for multiplication using the Desmos Graphing Calculator.

A

Assess Reasonableness. Use of strategies (e.g., estimation) to ensure an answer makes sense or is reasonable in the context of a given problem.

Jamie won a jar of 273 lollipops in a raffle. He wants to share them equally with his classmates. There are 24 total students in Jamie's class. How many lollipops will each student get?

$$273 \div 24 = ?$$

Estimate... $275 \div 25 = 11$. I used two friendlier numbers very close to the numbers in the problem, so my answer will be about 11. My solution is each student will get 11 lollipops and there will be 9 leftovers. My answer is reasonable.

Examples:

How can estimation and numerical reasoning, along with the product of 24×63 be used to compute the answer to 24×0.63 .

$$24 \times 63 = 1512$$

Estimate... 24×0.63 is a bit more than $\frac{1}{2}$ of 24. $\frac{1}{2}$ of 24 is 12. So, the decimal goes between the 5 and the 1 in 1512 because 15 is a bit more than 12.

A

Assessment. The evaluation or estimation of the ability of someone.

Assessment Examples

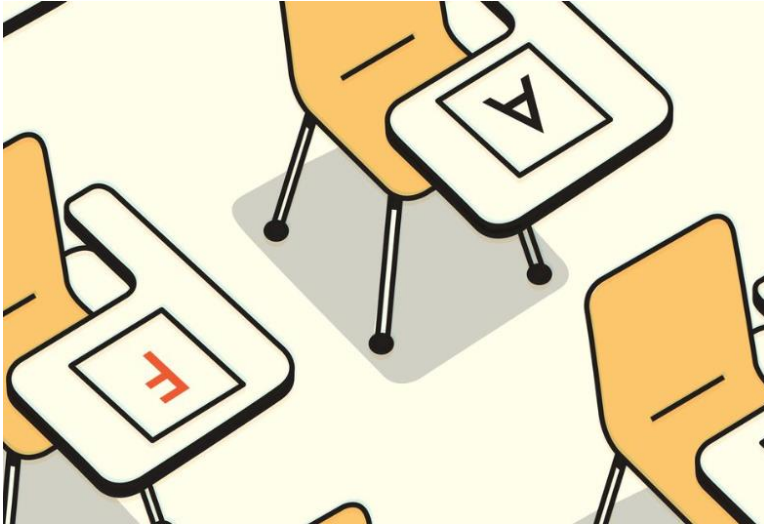


Image source: <https://www.edutopia.org/>

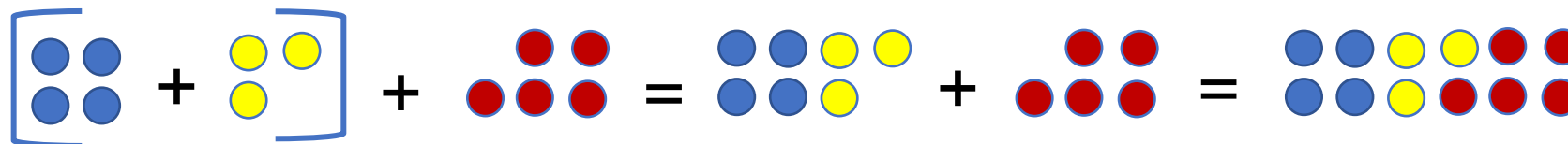
Formative Instructional Practices (FIP)

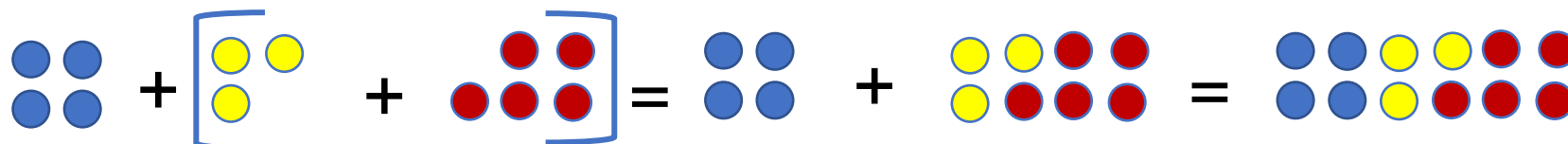


A

Associative Property of Addition. The associative property of addition states that how the numbers in an addition problem are grouped doesn't change the sum. To visualize this, take a look at the images below. There are three sets of dots with different colors. The dots being added are identical and the different colors represent the addends. See also: [Table 3 in this Glossary](#).

Example:


$$(4 + 3) + 5 = 7 + 5 = 12$$


$$4 + (3 + 5) = 4 + 8 = 12$$

A

Associative Property of Multiplication. The associative property of multiplication states that how the numbers in a multiplication problem are grouped doesn't change the product. To visualize this, take a look at the images below. See *a/so*: [Table 3 in this Glossary](#).

Example:

$$(2 \times 3) \times 4 = 2 \times (3 \times 4)$$

(2 groups of 3) groups of 4 2 groups of (3 groups of 4)

$$(6) \times 4 = 2 \times (12)$$

(6) groups of 4



is the same quantity as

2 groups of (12)



24

=

24

A

Automaticity. The ability to do something without occupying the mind with the low-level details required, allowing it to become an automatic response, pattern, or habit. Automaticity is the result of learning with deep understanding and purposeful practice. In mathematics, automaticity with math facts occurs when students build strategies for computation based on ideas such as estimation, place value, properties of operations and relational thinking. Purposeful practice of these ideas includes using developed strategies to solve contextual problems.

Automaticity:

- Efficient and accurate recognition of mathematics facts and terminology
- Frees cognitive resources to process meaning over the mechanics of computation
- Is achieved through purposeful practice of efficient and effective strategies

B

[Base \(Geometry\)](#)

[Base \(Number\)](#)

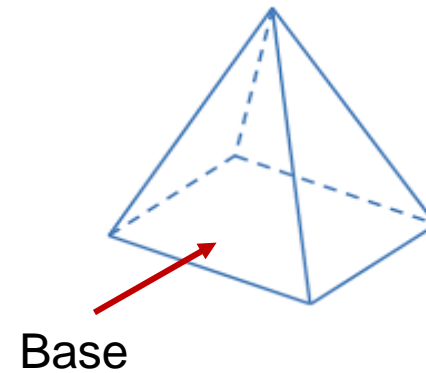
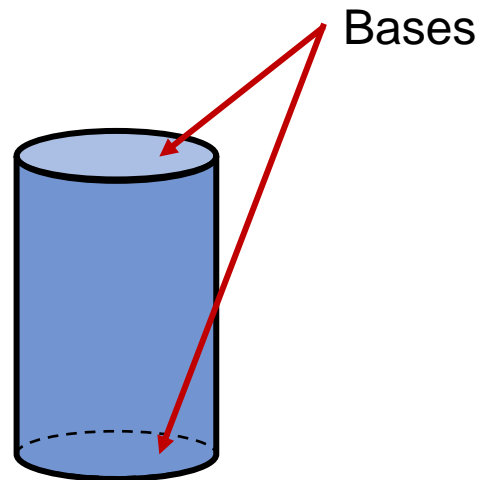
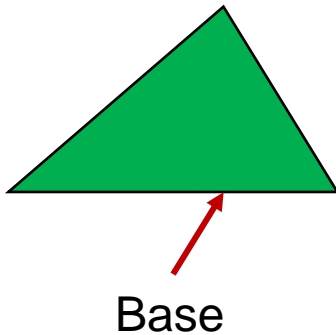
[Bivariate Data](#)

[Box Plot](#)

B

Base (Geometry). One side of a polygon, usually used as a reference side for other measurements. One or two faces of a solid are sometimes called a base. See *also*: [Polygon](#)

Examples:



B

Base (Number). A number to be raised to a power, denoted by an exponent. Also, a set of digits used to represent numbers. See *also*: [Exponent](#), [Place Value](#)

Examples:

$$4^3 = 4 \times 4 \times 4$$

Diagram illustrating the base and exponent in the equation $4^3 = 4 \times 4 \times 4$. The number 4 is labeled as the "base" and the number 3 is labeled as the "exponent or power".

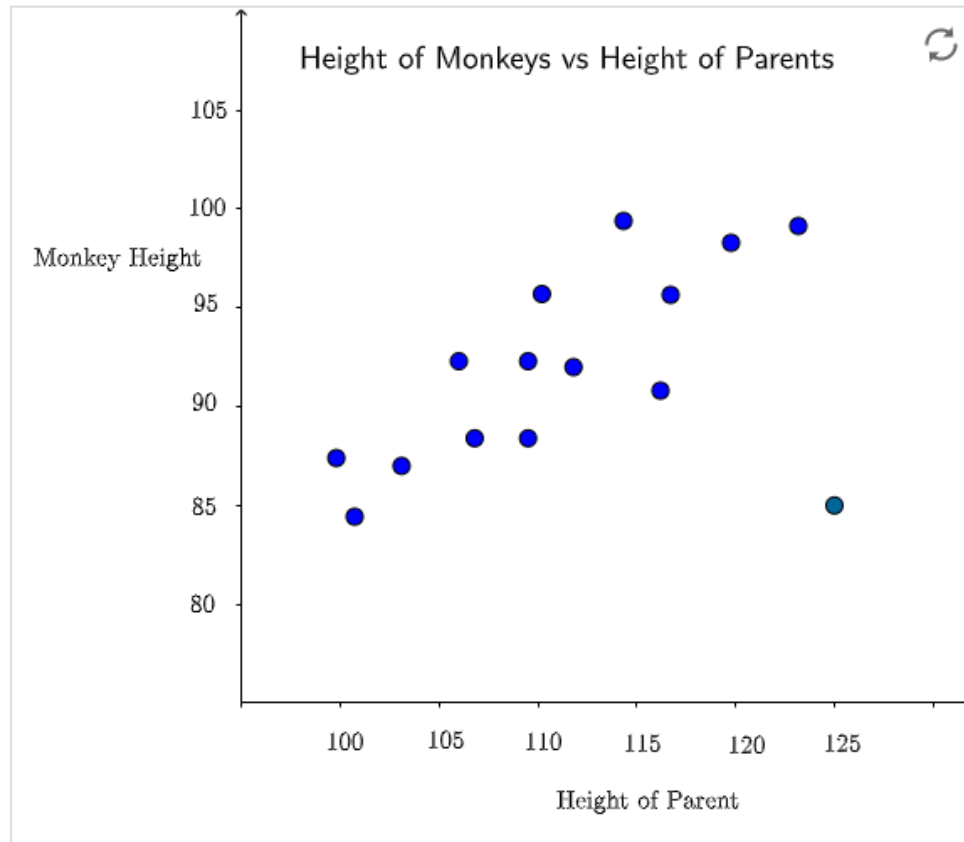
In the base-ten (decimal) number system, we use ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. These digits are used to represent any number in the base-ten (decimal) system.

The base-two (binary) system uses only two digits: 0 and 1, to represent all numbers in the base-two (binary) system.

B

Bivariate Data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Example:



Click the image to try an interactive scatter plot for bi-variate data using Geogebra.

B

Box Plot. A visual display of the distribution of data values using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.

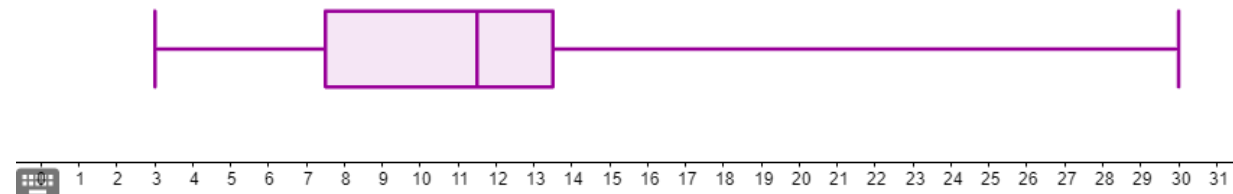
Example:

[Adapted from Wisconsin Department of Public Instruction](#)

Click the image to try an interactive box plot using GeoGebra.

The Box and Whisker Plot

- The highest score is:
- The lowest score is:
- The range is:
- Upper quartile Q_3 is:
- Lower quartile Q_1 is:
- Interquartile range (IQR) is:
- The median is:



	A
1	22
2	7
3	8
4	9
5	12
6	12
7	9
8	12
9	15
10	12
11	3
12	11
13	19
14	30
15	7
16	5
17	
18	

C

[Cardinality](#)

[Chord](#)

[Circle](#)

[Circular Cone](#)

[Circumference](#)

[Coefficient](#)

[Commutative Property of Addition](#)

[Commutative Property of Multiplication](#)

[Compare Problems](#)

[Complementary Angles](#)

[Complex Fraction](#)

[Complex Numbers](#)

[Composite Number](#)

[Composition/Decomposition of Number](#)

[Computation Algorithm](#)

[Computation Strategy](#)

[Conceptual Understanding](#)

[Cone](#)

[Congruent](#)

[Constant](#)

[Coordinates](#)

[Counting On](#)

[Cube](#)

[Cylinder](#)

C

Cardinality. The understanding that when you count items, the number word applied to the *last object counted* represents the total amount.



Example:

If a student is asked to count the bears to the left, they may count them correctly as “1, 2, 3, 4, 5, 6.”

When asked, again, how many bears there are, a student who has developed an understanding of cardinality will say, “6, there are 6 bears.”

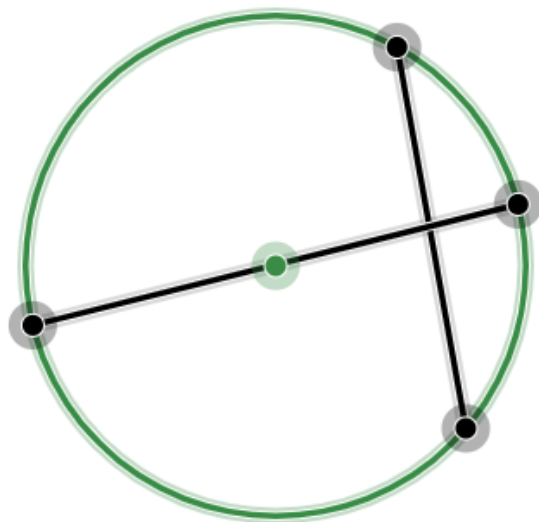
Students who have not, yet developed this understanding will count them all, again, “1, 2, 3, 4, 5, 6.”

C

Chord. A segment joining two points on a circle. See *also*: [Radius](#), [Diameter](#)

Example:

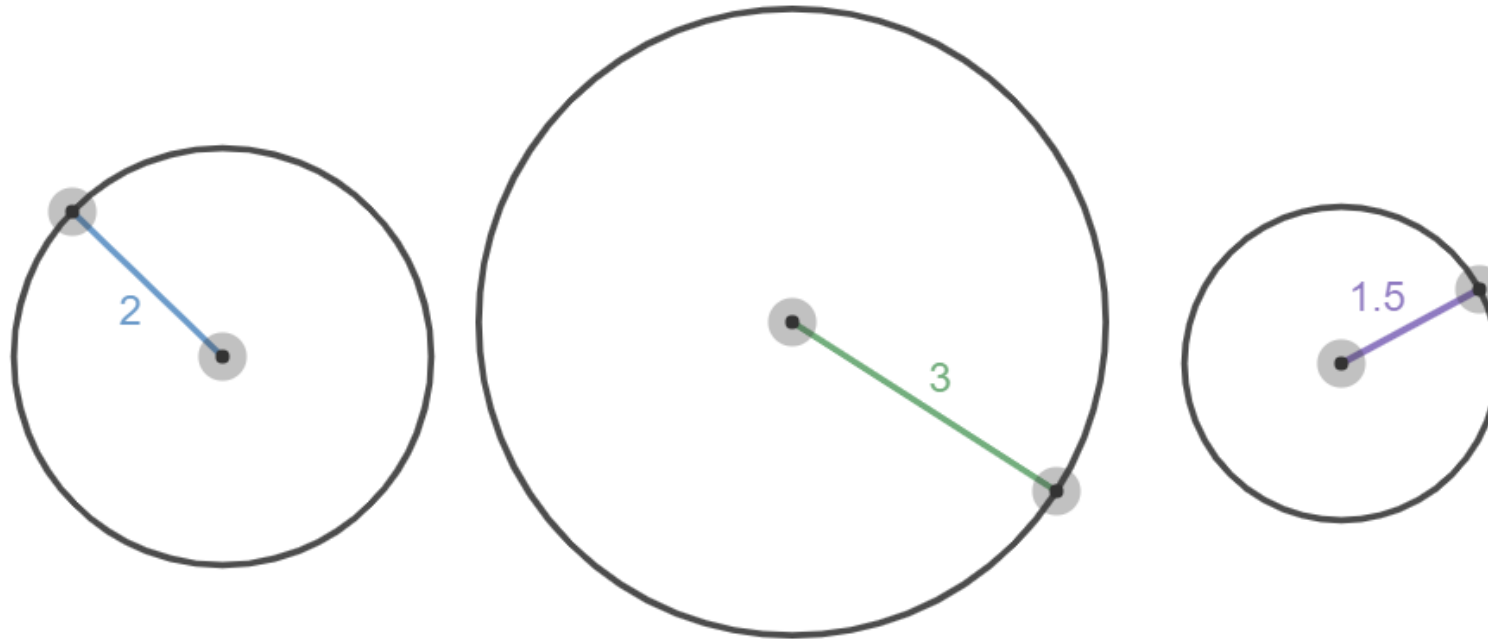
The black line segments in the image below show two chords in the green circle.



C

Circle. A continuous curved line whose points are all the same distance away from a fixed, central, point. See also: [Circumference](#), [Chord](#), [Diameter](#), [Radius](#).

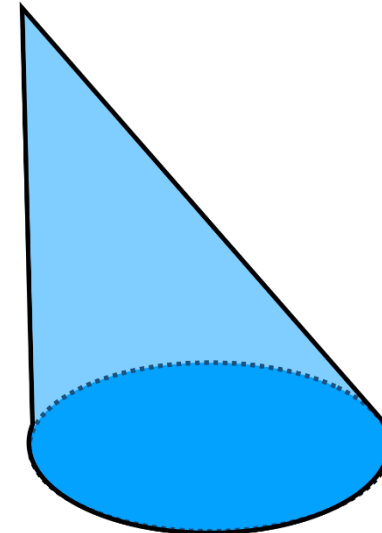
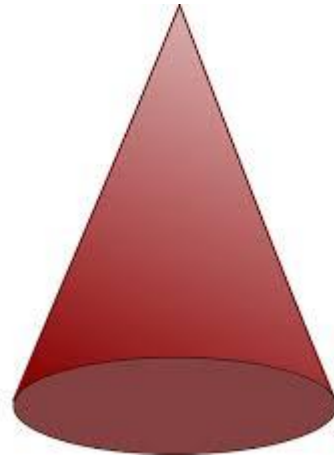
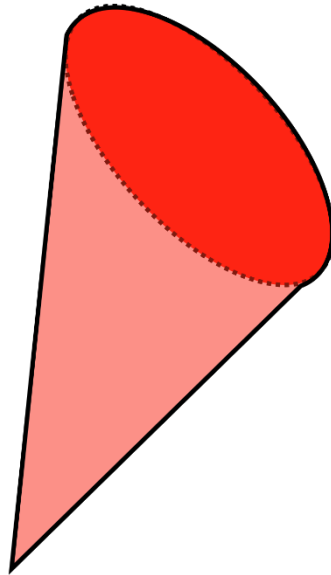
Examples:



C

Circular Cone. A cone with a circular base.

Examples:

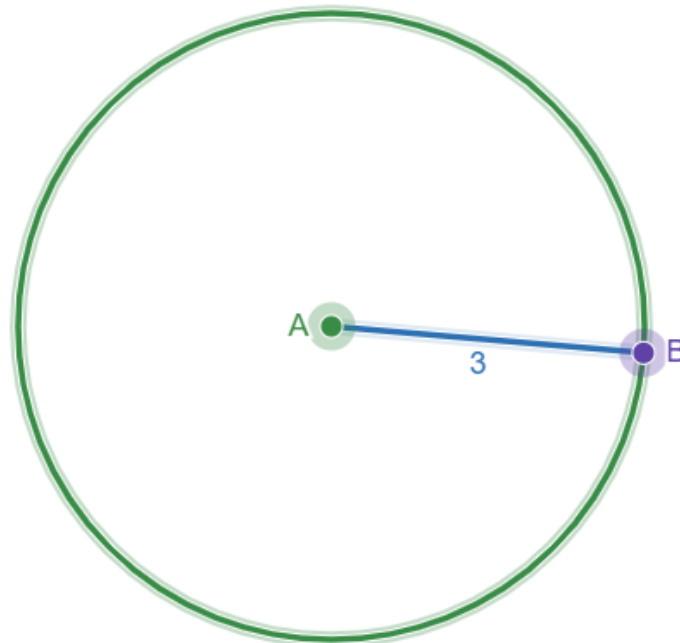


C

Circumference. The complete distance around a circle.

Example:

The circumference of the circle below is approximately 18.84 units.



C

Coefficient. A letter or number representing a numerical quantity that multiplies a term.

Examples:

Coefficient

$$-4x - 9 = 19$$

$$x(a + b)$$

Coefficient

Coefficients

$$ax + by + c = 12$$

$$3n$$

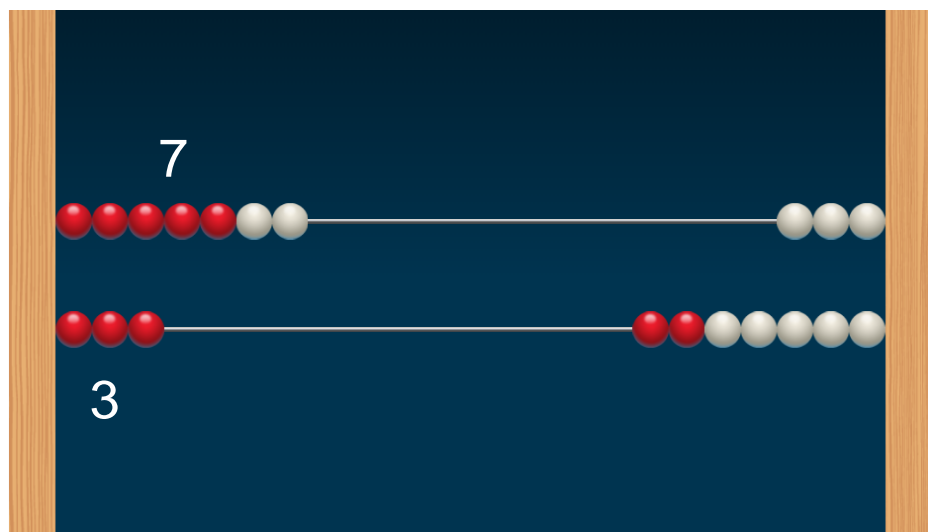
Coefficient

C

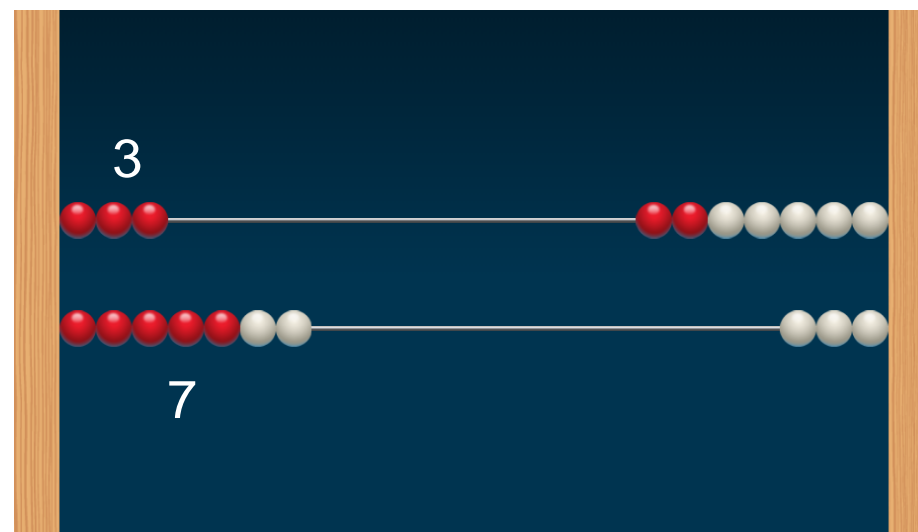
Commutative Property of Addition. The commutative property of addition states that the order in which two numbers are added does not change the sum. See also: [Table 3 in this Glossary](#).

Examples:

$$7 + 3 = 10$$



$$3 + 7 = 10$$

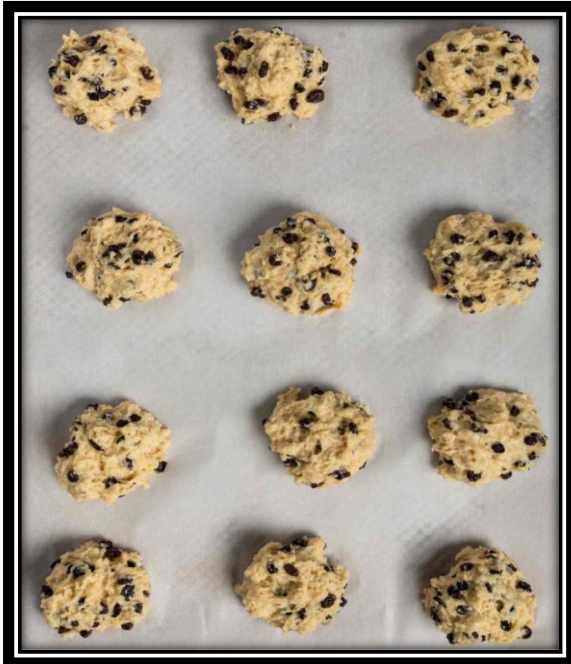


C

Commutative Property of Multiplication. The commutative property of multiplication states that the order in which two numbers are multiplied does not change the product. See also: [Table 3 in this Glossary](#)

$$4 \times 3 = 12$$

Four rows of three



Examples:

$$3 \times 4 = 12$$

Three rows of four



C

Compare Problems. Compare problems involve relationships between quantities rather than a joining and separating action. Compare problems involve the comparison of two distinct, disjoint sets. One of these sets is the “referent” set and the other is the “compared” set. See *also*: [Problem Types](#)

Examples:

Difference Unknown

Louis has 13 baseball cards. Jenna has 22 baseball cards. Jenna has how many more baseball cards than Louis?

Compared Set Unknown

Louis has 13 baseball cards. Jenna has nine more baseball cards than Louis. How many baseball cards does Jenna have?

Referent Set Unknown

Jenna has 22 baseball cards. She has nine more baseball cards than Louis. How many baseball cards does Louis have?

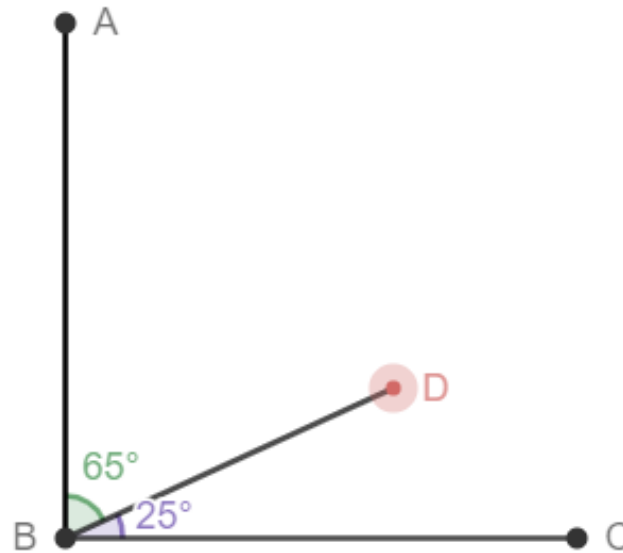
C

Complementary Angles. Two angles are complementary when their sum is 90° .

Example:

In the image below, $\angle ABD$ and $\angle DBC$ are complementary because $65^\circ + 25^\circ = 90^\circ$.

Click the image to the right to explore complementary angles using Desmos Geometry.



C

Complex Fraction. A fraction in which the numerator and/or the denominator contain fractions.

A fraction, $\frac{A}{B}$ where A and/or B are fractions ($B \neq 0$).

Example:

*A bakery uses $\frac{1}{6}$ of a bag of flour to make a batch of cakes. The bakery used $\frac{1}{2}$ of a bag yesterday.
How many batches of cakes did they make yesterday?*

This situation can be represented with the complex fraction:

$$\frac{\frac{1}{2}}{\frac{1}{6}}$$

C

Complex Number. A number with two components, a real number component and an imaginary number component. Complex numbers are written in the form $a + bi$, where a is the real part and bi , is the imaginary part. See also: [Imaginary Numbers](#)

Example:

$$4 + 12i$$

Real number
component

Imaginary number
component

Note: The 'plus' sign does not mean 'add.' When you see a complex number like the one above, the plus sign is only used to separate the real and imaginary components.

C

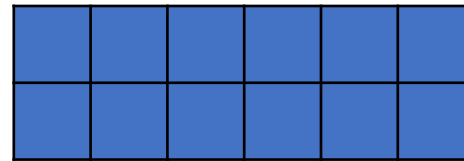
Composite Number. A positive integer that has more than two factors.

Examples:

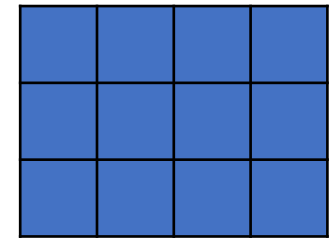
12 is a composite number because I can make 3 different rectangular arrays (6 if you count the 90° rotations of each). These give me all six factors of 12: {1, 2, 3, 4, 6, 12}.



1 x 12



2 x 6



3 x 4

7 is not a composite number because I can make only one rectangular array (2 if you count the 90° rotation of this array). This tells me that there are only two factors for 7: {1, 7}.



1 x 7

C

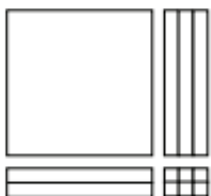
Composition/Decomposition of Number. A method of expressing a number in terms of its simpler components or of combining components of numbers in order to simplify computation.

Examples:



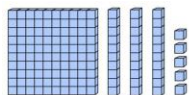
Rectangle 12×13

$$100 + 20 + 30 + 6 = 156$$

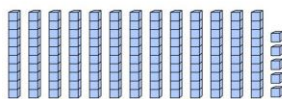


	10	+ 3
10	100	30
+ 2	20	6

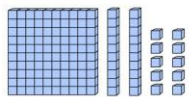
Rectangle not drawn to scale.



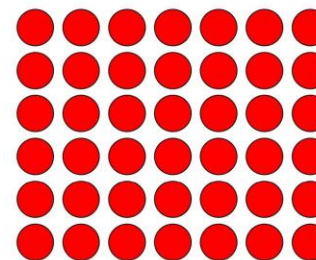
$$100 + 30 + 5 = 135$$



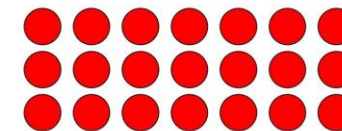
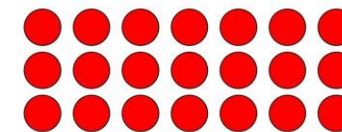
$$130 + 5 = 135$$



$$100 + 20 + 15 = 135$$



$$6 \times 7$$



$$(3 \times 7) + (3 \times 7)$$

$$\frac{11}{7} = \frac{7}{7} + \frac{4}{7} = 1 \frac{4}{7}$$

C

Computation Algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: [Computation Strategy](#).

Examples:

Subtraction Algorithm

$$\begin{array}{r} 401 \\ -189 \\ \hline \end{array} \xrightarrow{-2} \begin{array}{r} 399 \\ -187 \\ \hline 212 \end{array}$$

Addition Algorithm

$$\begin{array}{r} 438 \\ +287 \\ \hline 600 \\ 110 \\ +15 \\ \hline 725 \end{array} \quad \begin{array}{l} (400 + 200) \\ (30 + 80) \\ (8 + 7) \end{array}$$

Multiplication Algorithm

$$\begin{array}{r} 43 \\ \times 29 \\ \hline 800 \\ 360 \\ 60 \\ +27 \\ \hline 1247 \end{array} \quad \begin{array}{l} (40 \times 20) \\ (40 \times 9) \\ (20 \times 3) \\ (9 \times 3) \end{array}$$

Division Algorithm

$$\begin{array}{r} 4 \\ 2 \text{ } \textcircled{\cdot} \\ 1 \text{ } \textcircled{\cdot} \text{ } \textcircled{\cdot} \\ \hline 7 \overline{) 868} \\ -7 \text{ } \textcircled{\cdot} \text{ } \textcircled{\cdot} \\ \hline 168 \\ -14 \text{ } \textcircled{\cdot} \\ \hline 28 \\ -28 \\ \hline 0 \end{array}$$

C

Computation Strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: [Computation Algorithm](#), [Computational Thinking Video](#)

Examples:

Subtraction Strategy

$$\begin{array}{r} 401 \xrightarrow{-2} 399 \\ -189 \xrightarrow{-2} -187 \\ \hline 212 \end{array}$$

Addition Strategy

$$\begin{array}{r} 438 \\ +287 \\ \hline 600 \quad (400 + 200) \\ 110 \quad (30 + 80) \\ + 15 \quad (8 + 7) \\ \hline 725 \end{array}$$

Multiplication Strategy

$$\begin{array}{r} 43 \\ \times 29 \\ \hline 800 \quad (40 \times 20) \\ 360 \quad (40 \times 9) \\ 60 \quad (20 \times 3) \\ + 27 \quad (9 \times 3) \\ \hline 1247 \end{array}$$

Division Strategy

$$\begin{array}{r} 4 \\ 1 \overline{) 868} \\ \underline{868} \\ 0 \end{array}$$

C

Conceptual Understanding. Understanding which ideas are critical to a mathematical concept, allowing for the use of ideas strategically and flexibly to solve problems (especially non-routine problems), thereby avoiding common misunderstandings.

Example: Fractions

In order for students to truly understand fractions and, eventually, how to operate with them, they need to have a firm grasp on the following big ideas of fractional understanding:

Part of a whole



Part of a set



Length



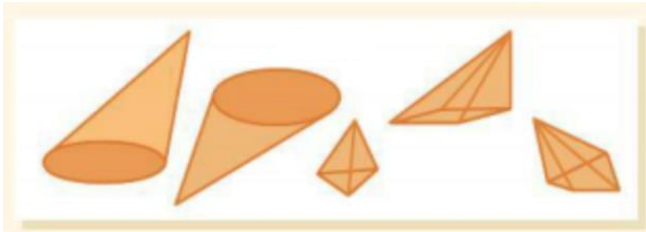
- Equal sharing is a way to build on whole-number knowledge to introduce fractional amounts.
- Students must experience fractions across many constructs, including part of a whole, division, and ratios.
- Three categories of models exist for working with fractions: area, length, and set or quantity.
- Partitioning and iterating are ways for students to understand the meaning of fractions, especially numerators and denominators.
- Equivalent fractions are ways of describing the same amount by using different-sized fractional parts.
- Fractions can be compared by reasoning about the relative size of the fractions. Estimation and reasoning are important in teaching understanding of fractions.

C

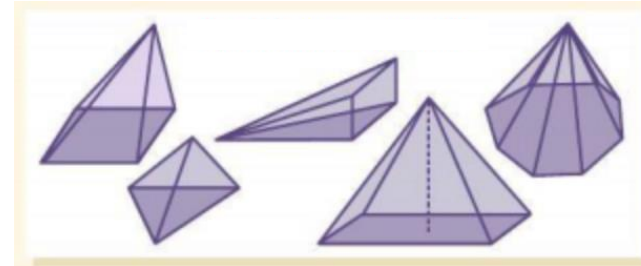
Cone. A solid with exactly one face and a vertex that is not on the face.

Examples:

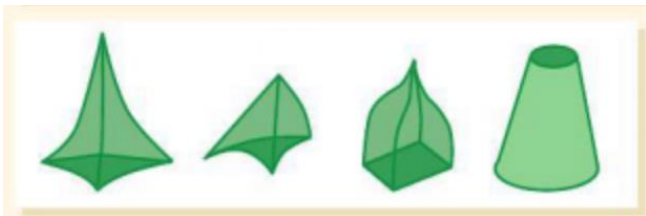
Cones



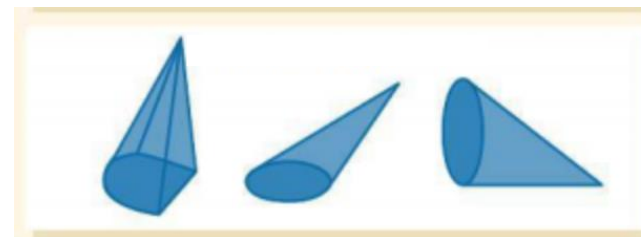
Special cones – pyramids



Not cones



Cones – not pyramids

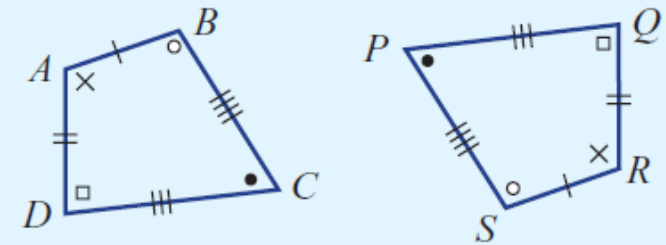


C

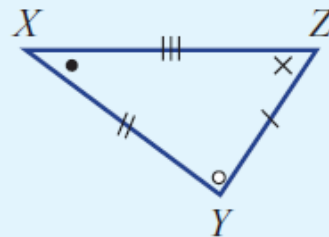
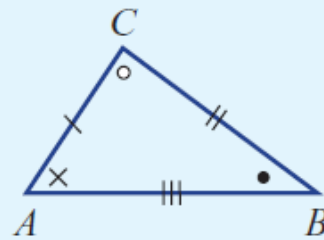
Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Click the image to try an interactive congruence lesson using Geogebra.

- **Congruent figures** have the same shape and size. They are identical in every way.
 - If two figures are congruent, one of them can be transformed by using rotation, reflection and/or translation to match the other figure exactly.



- In the above diagram, AB and RS are called ‘matching sides’ or ‘corresponding sides’.
- Consider these congruent triangles.



Matching (corresponding) sides	Matching (corresponding) angles
$AB = ZX$	$\angle A = \angle Z$
$BC = XY$	$\angle B = \angle X$
$AC = ZY$	$\angle C = \angle Y$

We write $\triangle ABC \cong \triangle ZXY$ (not $\triangle ABC \cong \triangle XYZ$)

C

Constant. A term with a fixed value in an expression or equation. A constant is not affected by any changes in the variable.

Examples:

$$9n + 8 = 37$$

↑
Constant

Constant

↓

$$-4x - 9 = 19$$

Constant

↓

$$3a - 4b + 7 = 12$$

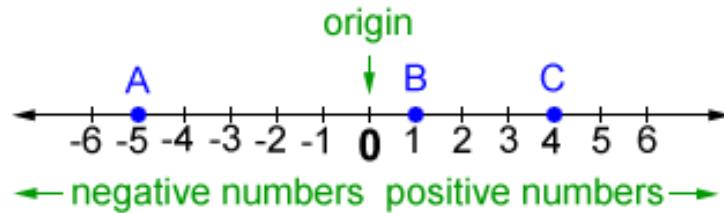
C

Coordinates. A set of numbers that specify the position of a point in a coordinate system. A real number that matches the location of a point along a number line is called a coordinate of that point.

See also: [x-axis](#), [x-coordinate](#), [y-axis](#), [y-coordinate](#), [z-axis](#), [z-coordinate](#)

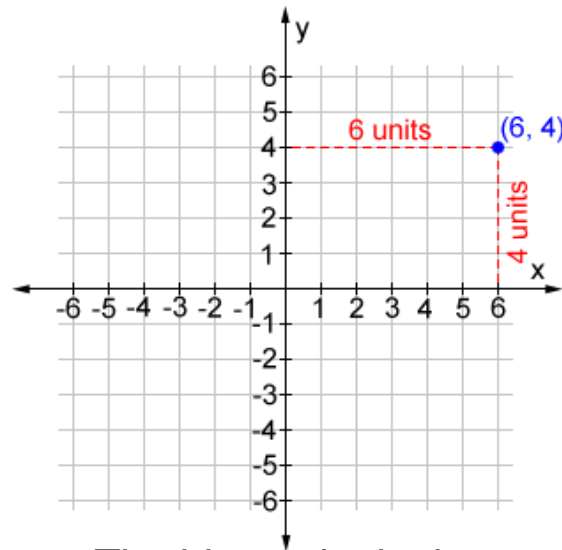
Examples:

1-Dimensional Coordinates



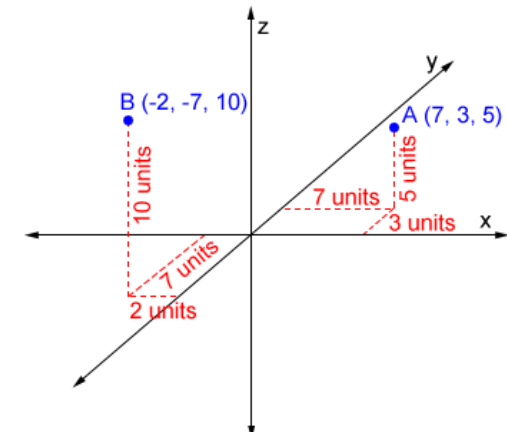
Point A has a coordinate of -5, point B has a coordinate of 1, and the coordinate for point C is 4.

2-Dimensional Coordinates



The blue point in the graph above is located by the ordered pair, (6, 4). 6 is the x-coordinate and 4 is the y-coordinate.

3-Dimensional Coordinates



Point B in the graph above is located by the ordered triple, (-2, -7, 10). -2 is the x-coordinate, -7 is the y-coordinate, and 10 is the z-coordinate.

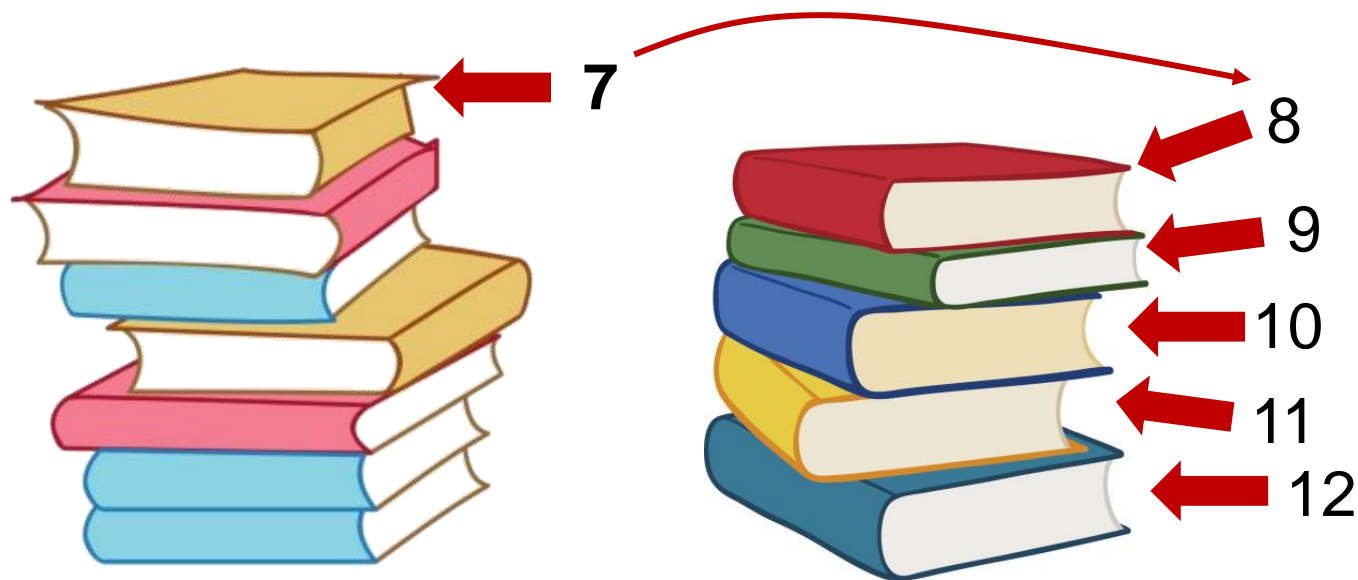
[Back to Start](#)

C

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group.

Example:

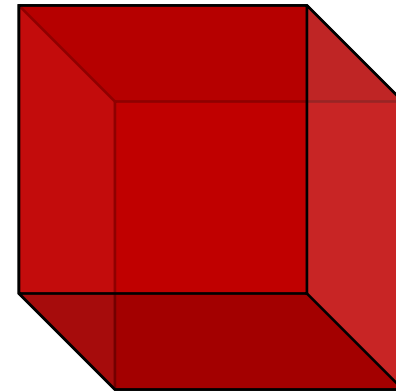
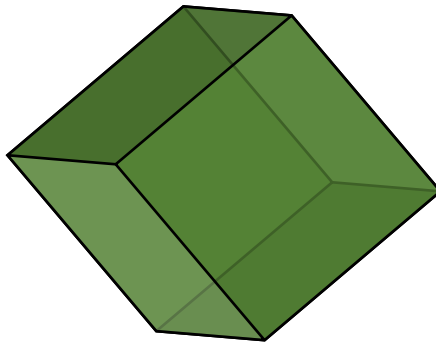
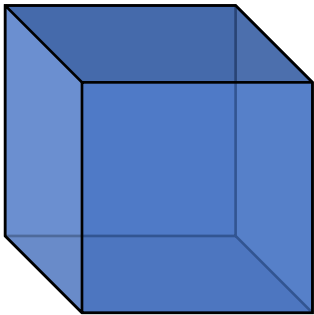
If a stack of books is known to have 7 books and 5 more books are added to the table, it is not necessary to count the first stack all over again. We can find the total by counting on—pointing to the top book and saying “seven,” following this with “eight, nine, ten, eleven, 12. There are twelve books now.”



C

Cube. A solid figure with 6 square faces.

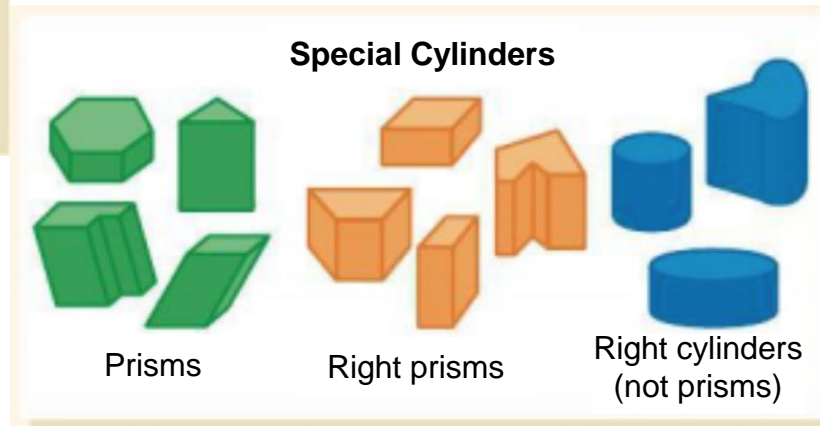
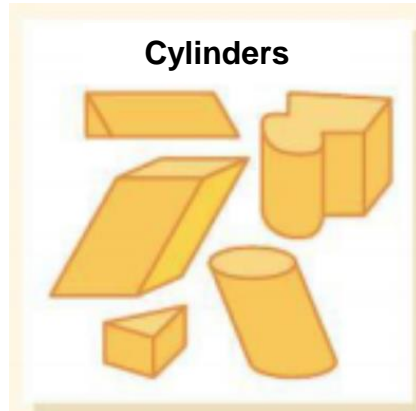
Examples:



C

Cylinder. Any three-dimensional figure with two congruent, opposite faces called bases connected by adjacent curved or flat surfaces (bases can include circles, triangles, rectangles, or other shapes). The bases can be connected by lines that are parallel to each other.

Examples:



D

[Decimal](#)

[Denominator](#)

[Diameter](#)

[Digit](#)

[Dilation](#)

[Domain](#)

[Dot plot](#)

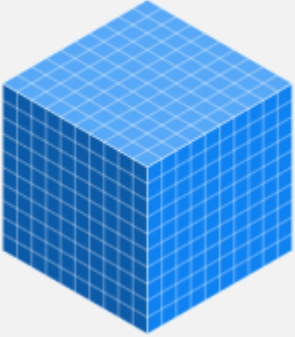
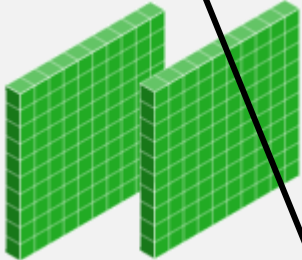
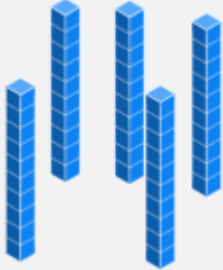

D


Decimal. A notation used for writing fractions within the base-ten system (using denominators of 10, 100, 1000, etc.). Decimals are also called Decimal Fractions. See *also*: [Repeating Decimal](#)
[Terminating Decimal](#)

Examples:

$$12.59 = 10 + 2 + \frac{5}{10} + \frac{9}{100}$$

Name of units

Tens	Ones	Tenths	Hundredths
			



The decimal point always “looks up at” the name of the units position. In this case, we have 12.59.

D

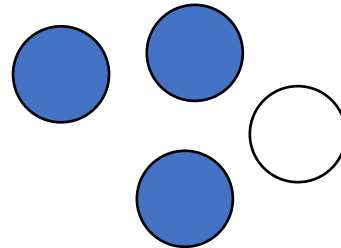
Denominator. In fraction notation, the denominator represents the number of parts in a whole. In the example below, the denominator, 4, represents the total number of parts in the whole. See *also*:

[Numerator](#), [Fraction](#)

Examples:

$\frac{3}{4}$

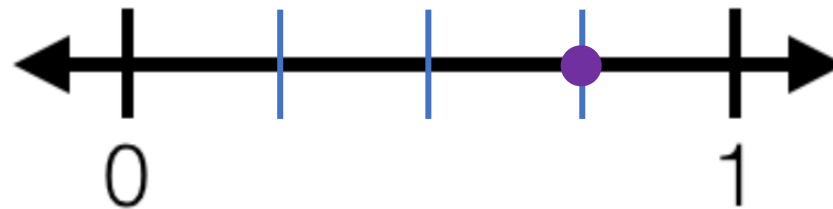
Denominator



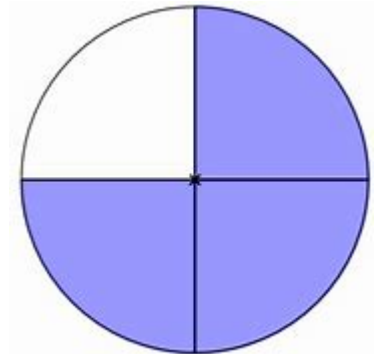
4 parts (pieces) in the whole set.



4 parts in the whole rectangle.



The whole is sectioned into 4 parts.



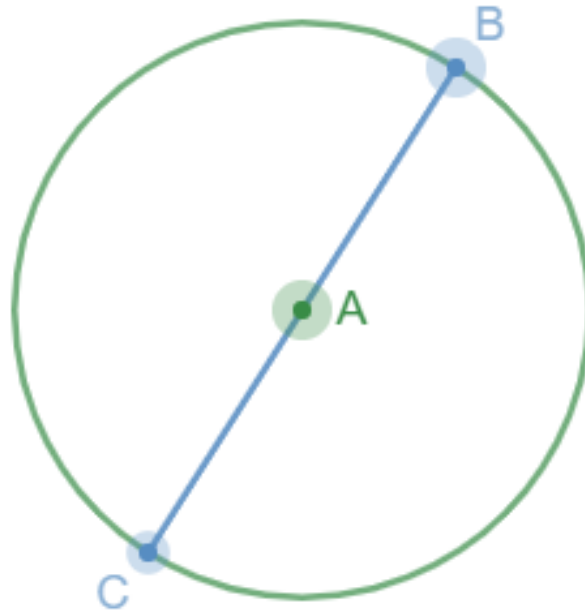
4 parts in the whole circle.

D

Diameter. A line segment that passes through the center of the circle and whose endpoints lie on the circle. A diameter of a circle is also the longest chord in a circle and divides the circle in half.

Example:

In the circle below, **line segment BC** is a diameter of **circle A** because its endpoints lie on the circle, and it passes through the center of the circle at point A.



D

Digit. A single symbol used to write numerals. See *Also:* [Number](#), [Numeral](#).

In our base-ten system, we have ten digits:

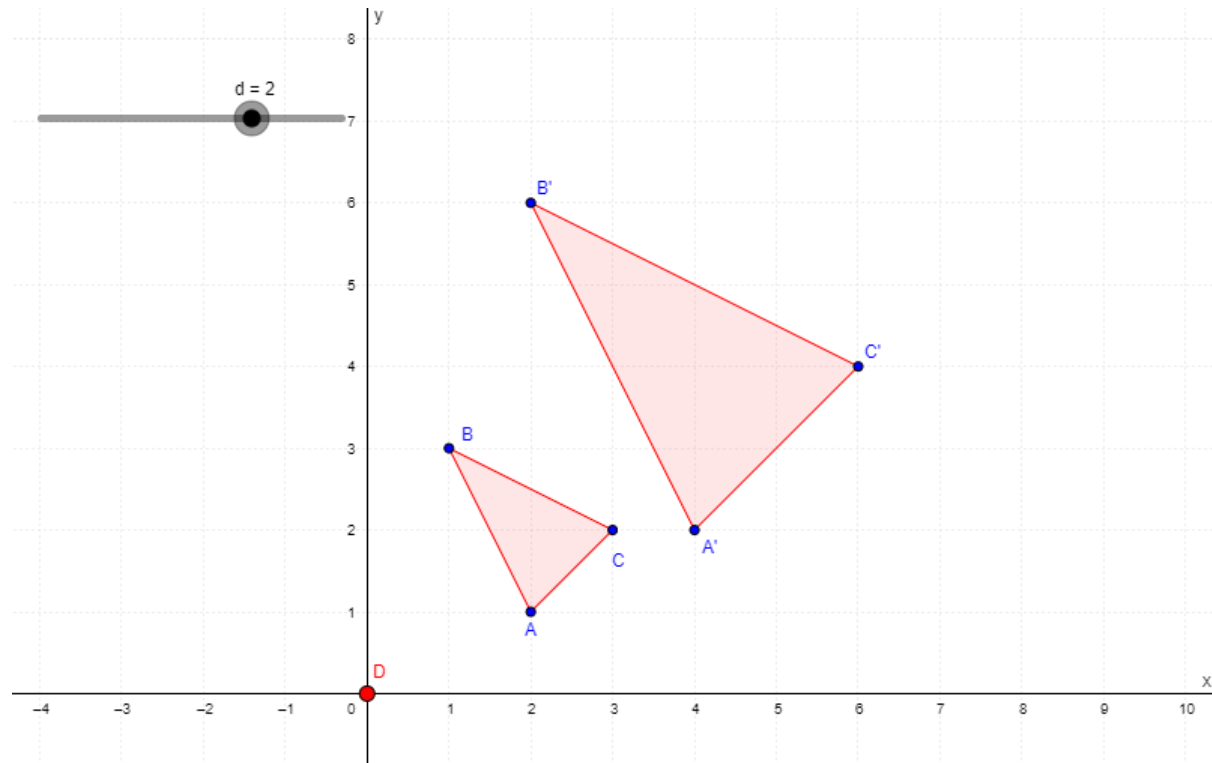
0, 1, 2, 3, 4, 5, 6, 7, 8, 9

D

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center and multiplies distances from the center by a common scale factor.

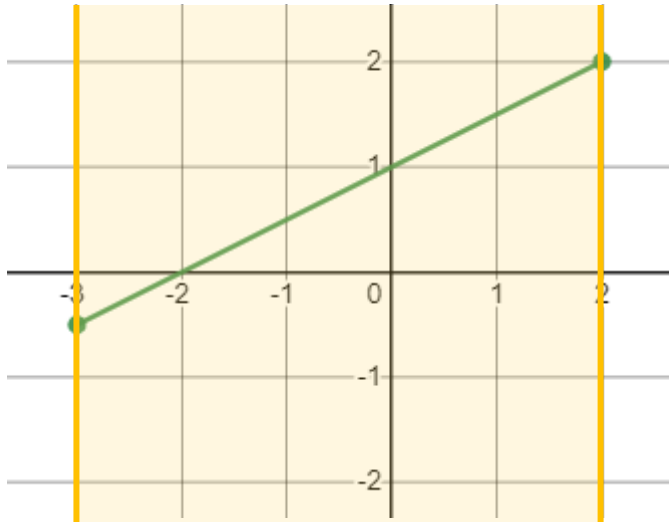
Example:

Click the image to try an interactive dilation tool using GeoGebra.

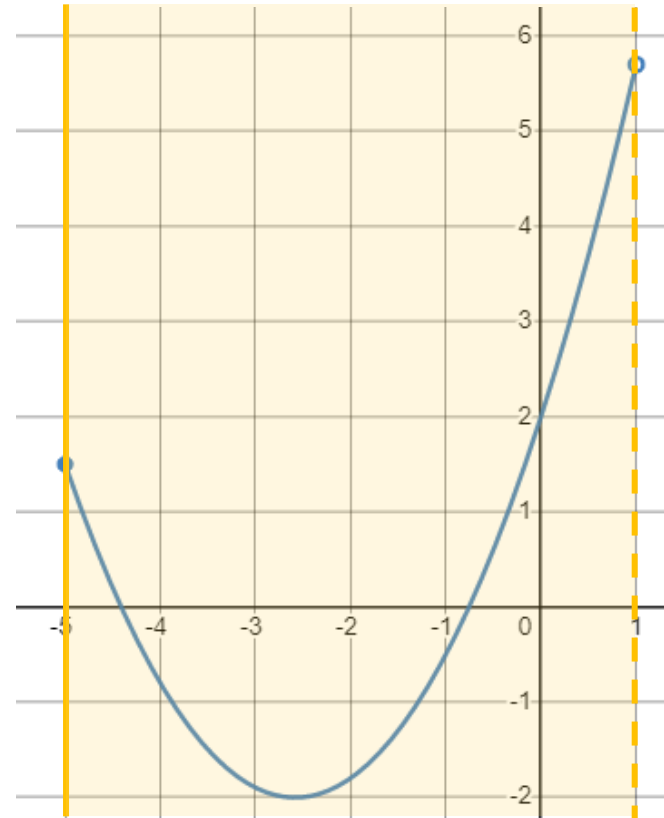


D

Domain. The set of input values for which the function is defined. See *also*: [Range](#)



The domain for this function
is $\{-3 \leq x \leq 2\}$



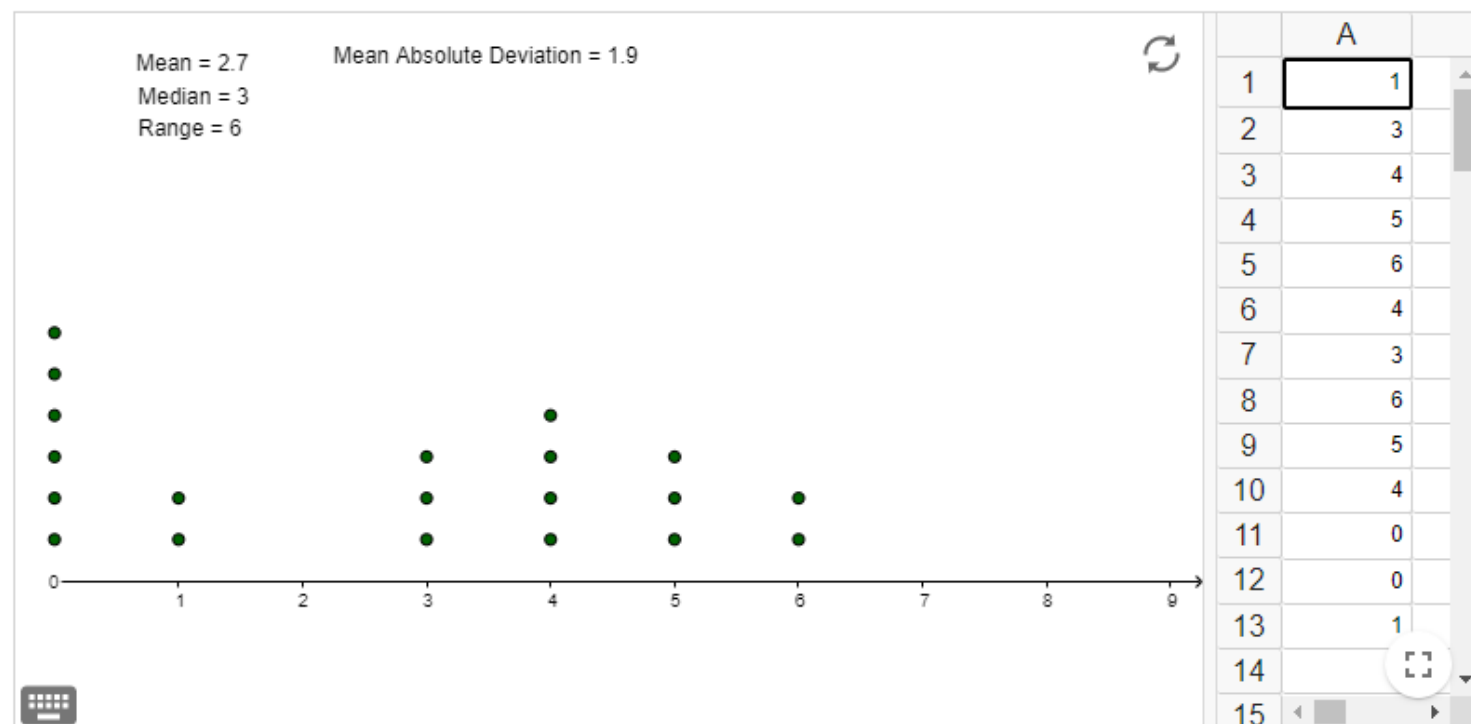
The domain for this function
is $\{-5 \leq x < 1\}$

D

Dot Plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a line plot.

Example:

Click the image to try an interactive dot plot tool using GeoGebra.



E

[Equation](#)

[Estimate](#)

[Expanded Form](#)

[Expected Value](#)

[Experiment](#)

[Exponent](#)

[Exponential Function](#)

[Expression](#)



[Back to Start](#)

E

Equation. A mathematical statement showing two expressions have the same value. See *also:* [Expression](#)

Examples:

Expression → $7 = 10 - 3$

Expression →

$$2\frac{3}{4} - \frac{1}{8} = n$$

$$12 + 8 = 10 + 10$$

$$5.34x + n = 24 - n$$

$$3x + 2 = y$$

E

Estimate. To estimate means to make an approximate calculation.

Example: About how many footprints will it take to measure the brick path?



The sense-making part of this task lies in the word “about.” Producing an estimate is a very difficult task for young children and requires a lot of time spent on helping them develop the idea of “about.” To help students develop this idea, instead of asking for a specific number, ask students whether the amount will be more or less than a target number. Some estimation question stems can be found below:

- Will it be more or less than _____?
- Will it be closer to _____ or to _____?
- Will it be about _____ (insert a benchmark of 5 or 10)?

E

Expanded Form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. Expanded form is often used to clarify the meaning of the base-ten representation. The expanded form of a number shows how it is composed of its base-ten units.

Example:

$$1643 = 1000 + 600 + 40 + 3$$

$$1643 = 1 \cdot 1000 + 6 \cdot 100 + 4 \cdot 10 + 3 \cdot 1$$

$$1643 = 1 \textit{ thousand} + 6 \textit{ hundreds} + 4 \textit{ tens} + 3 \textit{ ones}$$

E

Expected Value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

Example: Board game spinner

A board game uses the spinner shown below to determine how many spaces a player will move forward on each turn. The probability is $\frac{1}{2}$ that the player moves forward 1 space and moving forward 2 or 3 spaces each have $\frac{1}{4}$ probability.



$$\begin{aligned}\text{Expected value} &= \frac{1}{2} \cdot 1 \text{ space} + \frac{1}{4} \cdot 2 \text{ spaces} + \frac{1}{4} \cdot 3 \text{ spaces} \\ &= \frac{1}{2} + \frac{1}{2} + \frac{3}{4} \\ &= 1.75\end{aligned}$$

The expected value is 1.75 spaces.

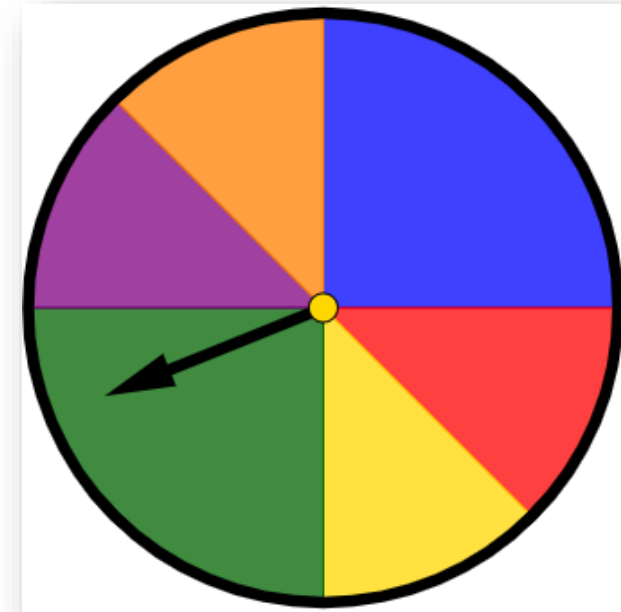
E

Experiment. Something that can be repeated that has a set of possible results.

Examples:

- Rolling dice to see what random numbers come up.
- Asking your friends what their favorite pet is and writing down the results.

Click the image to try an interactive experiment with a spinner using GeoGebra.



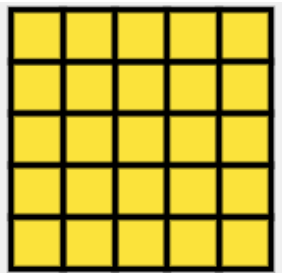
[Back to Start](#)

E

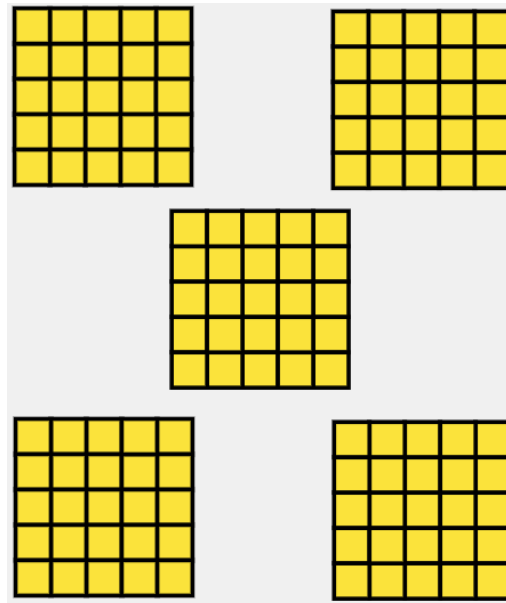
Exponent. A symbol written above and to the right of a mathematical expression to indicate the how many times the mathematical expression (base) should be multiplied by itself.

Examples:

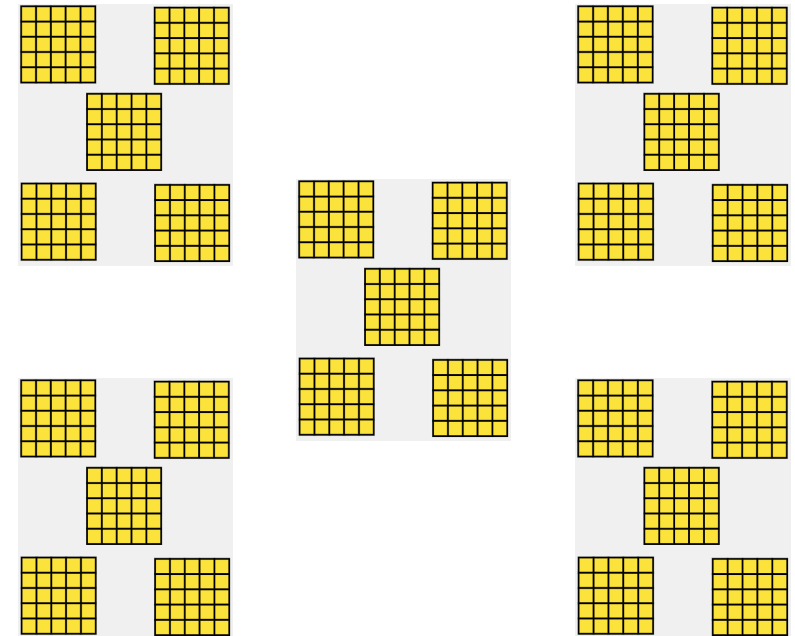
The array below shows
5 rows of 5 or 5×5 .
This can be written with
exponents as 5^2



The same 5×5 array, repeated
5 times, can be thought of as
5 groups of 5×5 and can be
represented as $5 \times 5 \times 5$ OR 5^3



The example to the left, repeated five times,
can be thought of as 5 groups of $5 \times 5 \times 5$
and can be represented as $5 \times 5 \times 5 \times 5$ or 5^4

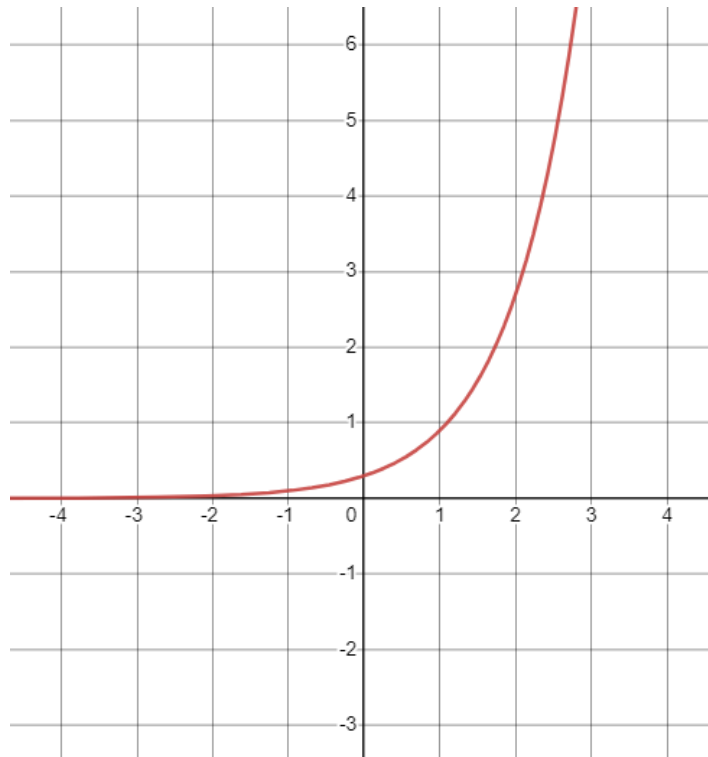


[Back to Start](#)

E

Exponential Function. An exponential function is a Mathematical function in the form $f(x) = ab^x$, where “x” is a variable “a” is a constant and “b” is a constant which is called the base of the function and should be greater than 0 and not equal to 1.

Click the image to explore an interactive exponential function graph using Desmos.



E

Expression. Mathematical statements that have a minimum of two terms containing numbers, variables, or both. These terms are connected by an operation such as addition, subtraction, multiplication, or division. See also: [Equation](#)

Example:

The diagram shows the algebraic expression $\frac{2}{3}x - 18$. Three red arrows point from labels to parts of the expression: a green arrow labeled "Coefficient" points to the fraction $\frac{2}{3}$; an orange arrow labeled "Variable" points to the letter x ; and a blue arrow labeled "Constant" points to the number 18.

F

[Factor](#)

[First Quartile](#)

[Fluency](#)

[Fraction](#)

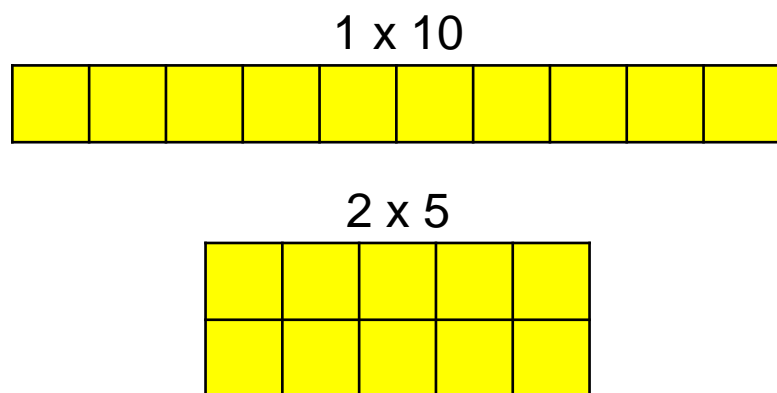
[Function](#)

F

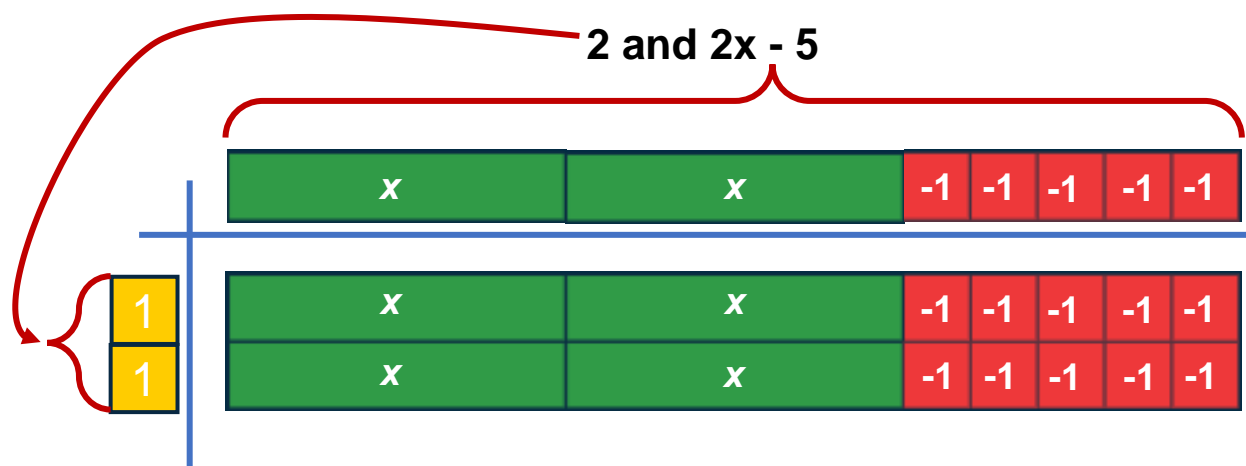
Factor. The numbers and/or variables that are multiplied to make a product or term.

Examples:

The number 10 has four factors: {1, 2, 5, 10}
This can be shown by building all possible arrays with ten tiles as shown below
(Rotations of each array are not shown due to space).



Factors of the expression $4x - 10$ can be found using algebra tiles. The factors are:



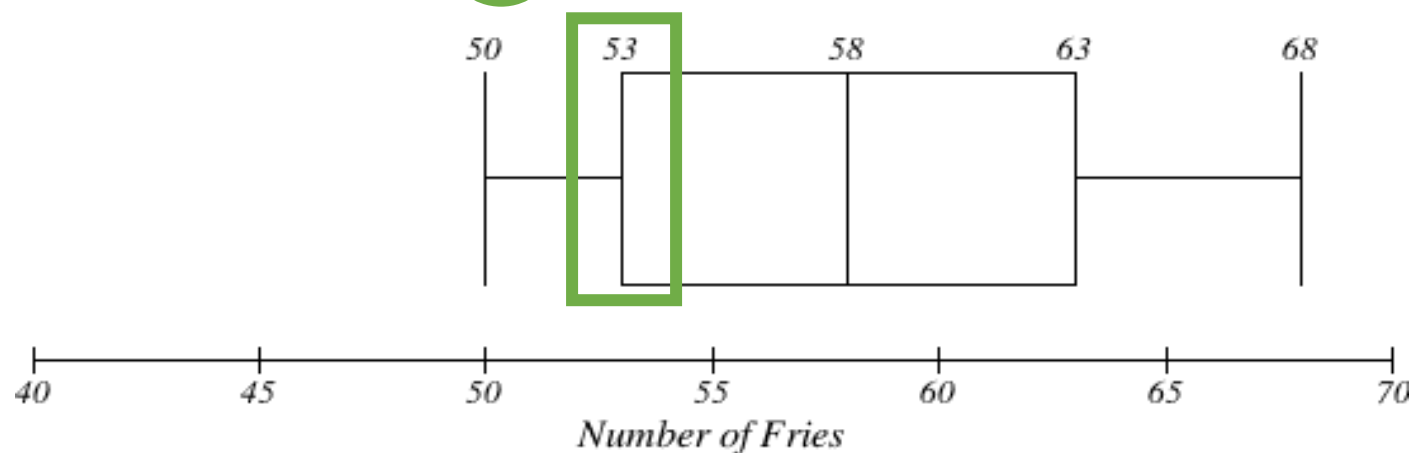
F

First Quartile. For a data set with median M , the first quartile is the median of the data values between M and the lower extreme; also called *Lower Quartile*. See also: [Median](#), [Third Quartile](#), [Interquartile Range](#), [Quartile](#).

Example: For the data set of the number of french fries in a small order of McDonalds fries, the first quartile is 53.

Number of French Fries in a Small Order of McDonalds Fries

50, 53, 53, 54, 56, 58, 59, 61, 63, 65, 68



F

Fluency. Fluency includes four ideas: efficiency, accuracy, flexibility, and the ability to choose appropriate procedures effectively:

Efficiency implies that the student does not get bogged down in too many steps or lose track of the logic of the strategy. An efficient strategy is one that the student can carry out easily, keeping track of subproblems and make use of intermediate results to solve the problem.

Accuracy depends on several aspects of the problem-solving process, among them careful recording, knowledge of number facts and other important number relationships, as well as double-checking results.

Flexibility requires the knowledge of more than one approach to solving a particular kind of problem, such as two-digit multiplication. Students need to be flexible in order to choose an appropriate strategy for the problem at hand, and also to use one method to solve a problem and another method to double-check the results. Fluency demands more of students than does memorization of a single procedure.

Appropriateness is the ability of students to know when to use a particular strategy or procedure.



For more information, see: [Fluency Without Fear](#)

[Back to Start](#)

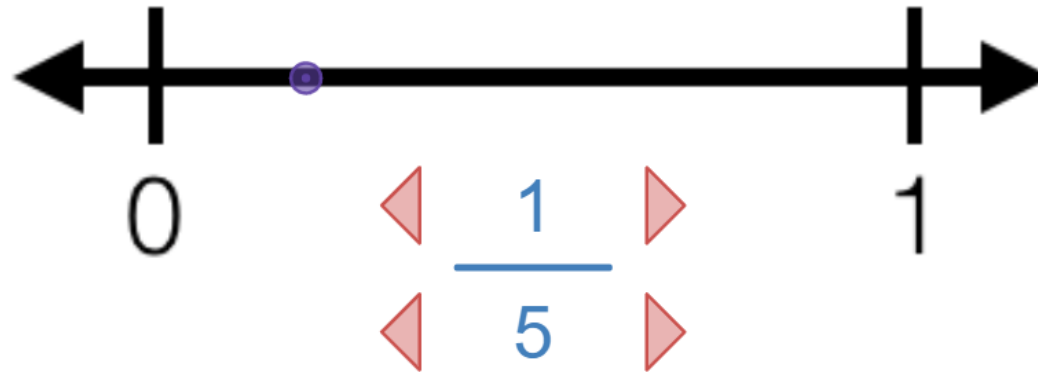
F

Fraction. A fraction represents a part of a whole, or more generally, any number of equal parts. Fraction notation includes two parts: a numerator and a denominator.

Examples:



Click the image to try an interactive fraction tool using Desmos.

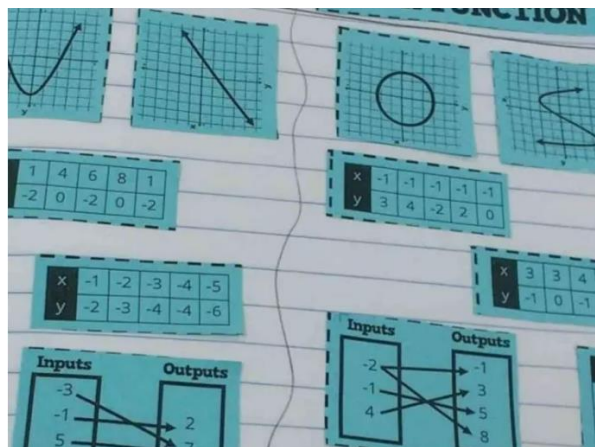


F

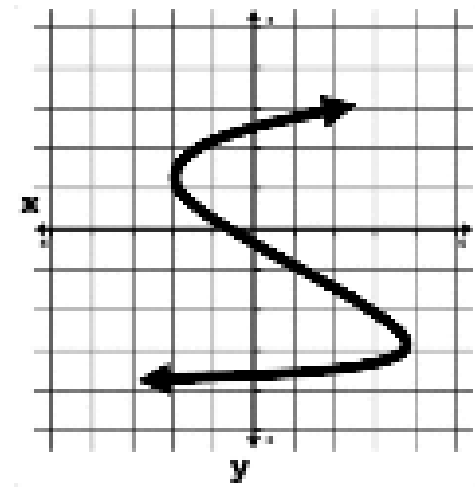
Function. A relation between a set of inputs where each input is related to exactly one output.

Examples:

Click the image below for a hands-on function activity from [Math for Love](#).



Click the image below for an interactive function activity using Desmos.



G

[Graph \(Algebra\)](#)

[Graph \(Statistics\)](#)

[Groups](#)

[Groups Problems](#)



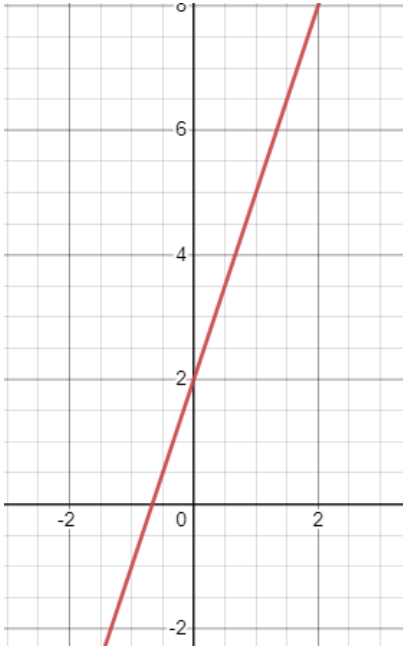
[Back to Start](#)

G

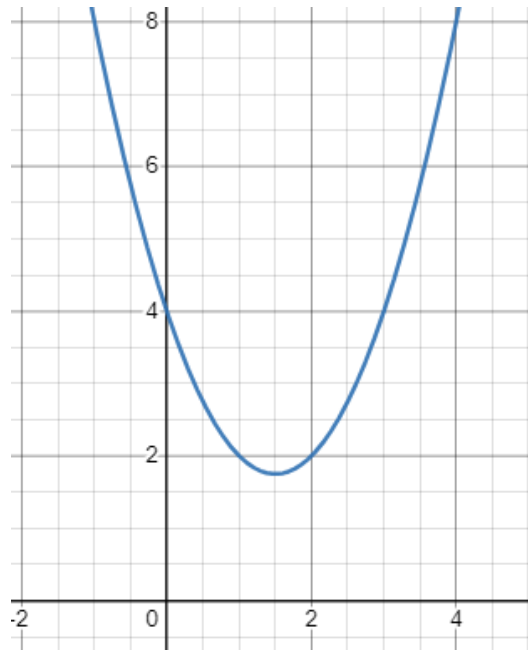
Graph (Algebra). A curve or line showing a mathematical function or equation, typically drawn in a Cartesian coordinate system.

Examples:

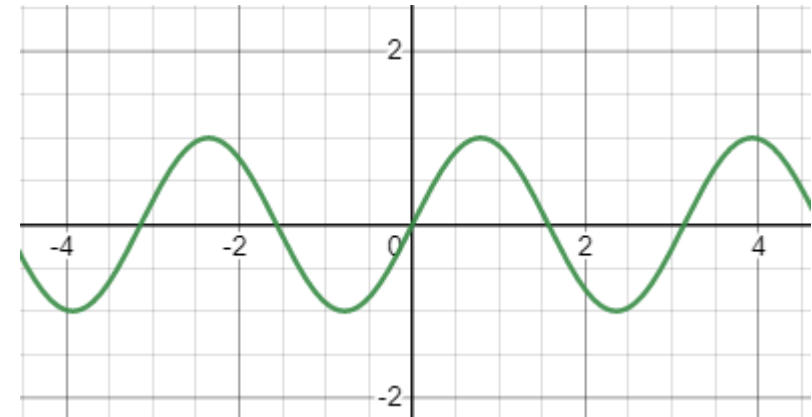
$$f(x) = 3x + 2$$



$$g(x) = x^2 - 3x + 4$$



$$h(x) = \sin 2x$$



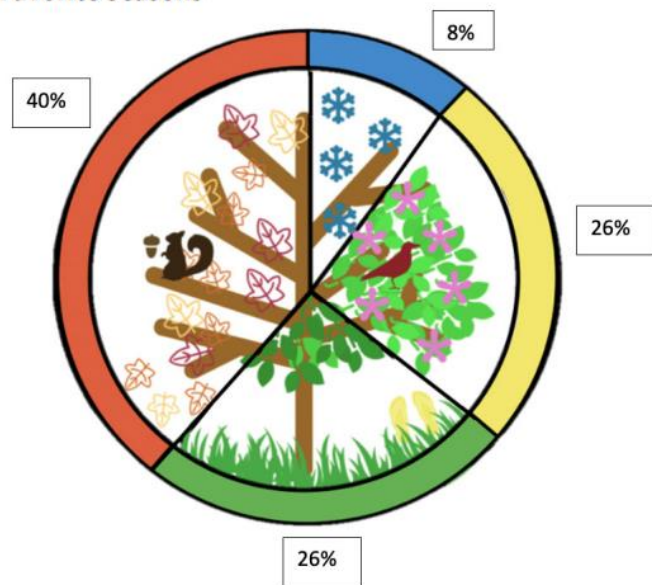
G

Graph (Statistics). A pictorial representation that represents data in an organized manner.

Examples:

[Click an image to learn more about data talks.](#)

Favorite Seasons

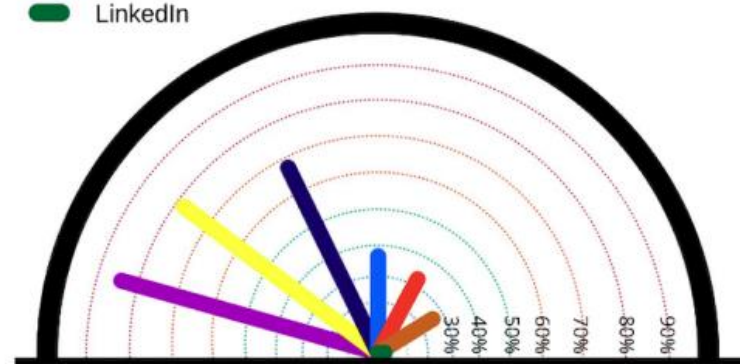


https://www.ipsos.com/sites/default/files/news_and_polls/2015-10/7014rev-topline.pdf

7 in 10 American Teens Use TikTok

% of U.S. teens who use the following social media platforms at least once a month

- Instagram
- Snapchat
- TikTok
- Twitter
- Pinterest
- Facebook
- LinkedIn

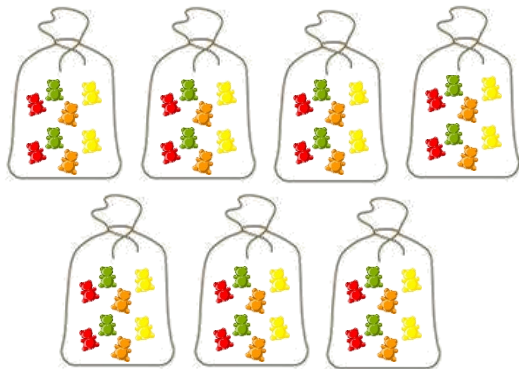


G

Groups. In multiplication, $A \times B$ or $A \cdot B$ is read as “A times B,” and means the total number of objects in A groups if there are B objects in each group. Shorthand: $A \times B$ means the total in A groups of B.

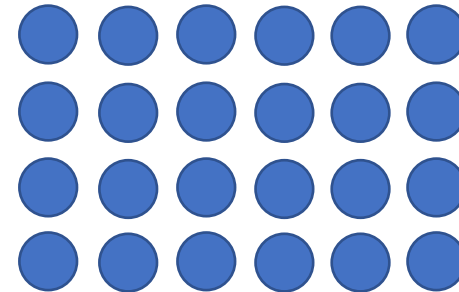
Examples:

*7 treat bags each contain 8 treats each.
How many total treats are in the seven
bags together?*



7×8 (7 groups of 8)

*How many soft drink cans in a case with 4
rows of 6 cans in each row?*



*Each row or column can be thought of as a group.
 4×6 (4 groups of 6)*

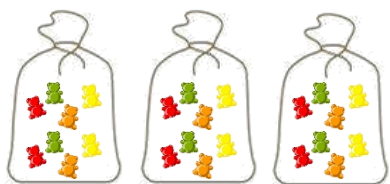
G

Groups Problems. When beginning the study of multiplication and division, problem types involve quantities that can be grouped or partitioned into equivalent groups with no leftovers or remainders. These grouping and partitioning problems all involve three quantities. See also, [Problem Types](#)

Examples:

Multiplication problems give the number of groups and the number of objects in each group. The unknown is the total number of objects.

Juan has three small bags of candy. Each bag has 8 candies in it. How many candies does Juan have?



$$3 \times 8 = ?$$

Measurement division problems provide the total number of objects and the number of objects in each group. The unknown is the number of groups. Children use the number of objects in each group to **measure** the total number of objects.

Juan has 24 candies. There are 8 candies in each bag. How many bags does he have?



$$24 \div 8 = ?$$

Partitive division problems give the total number of objects and the number of groups. The number of objects in each group is the unknown. Children **partition** the total number of objects into the given number of groups.

Juan has three bags of candies with the same number of candies in each bag. There are a total of 24 candies. How many candies are in each bag?



$$24 \div 3 = ?$$

H

[Hexagon](#)

[Hierarchical Inclusion](#)

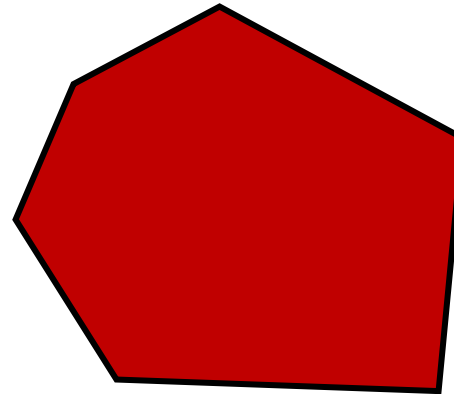
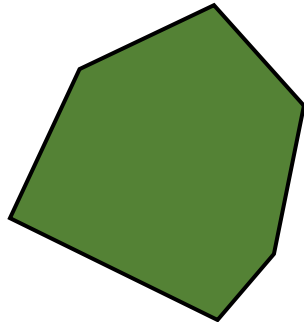
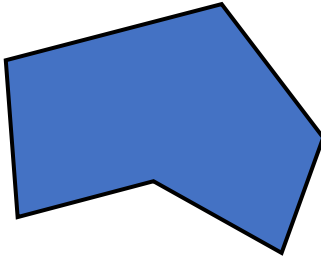
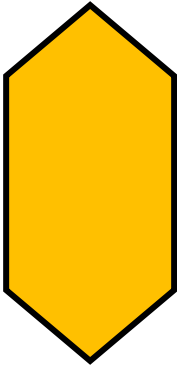
[Hierarchy](#)

[Hypotenuse](#)

H

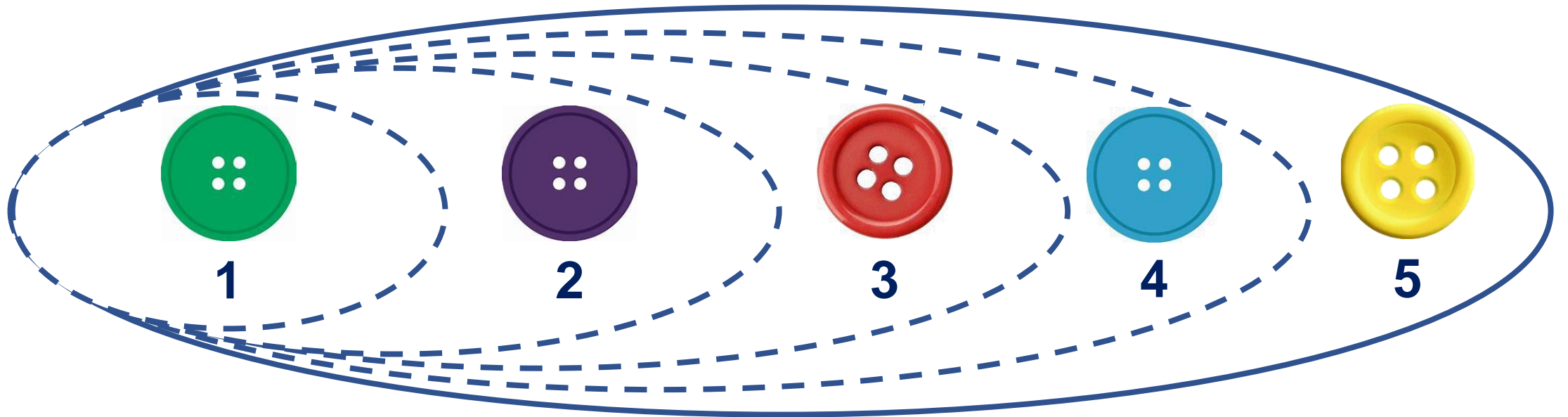
Hexagon. A six-sided and six-angled polygon. See *also*: [Polygon](#)

Examples:



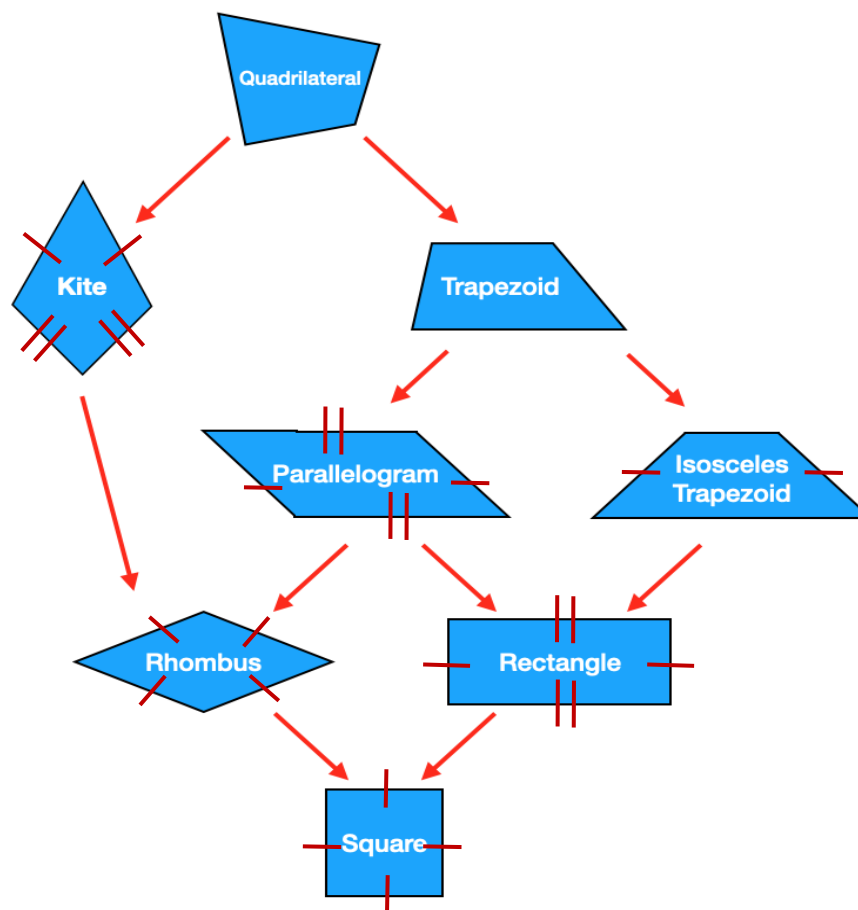
H

Hierarchical Inclusion. A counting principle based on the understanding that numbers are nested inside of each other. When children begin to make sense of hierarchical inclusion, they understand that as they count higher, the previous number is contained in the next highest number. Hierarchical inclusion is the first milestone in understanding [cardinality](#).



H

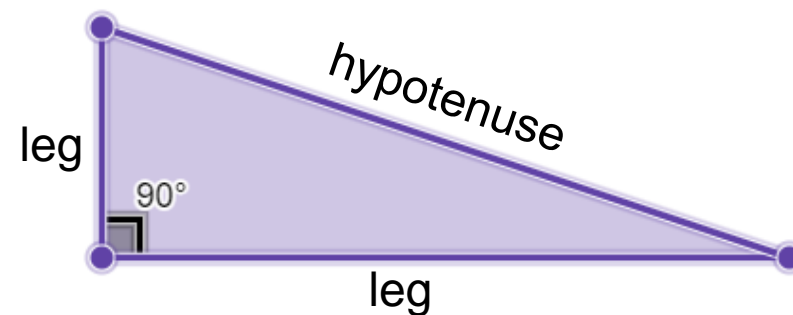
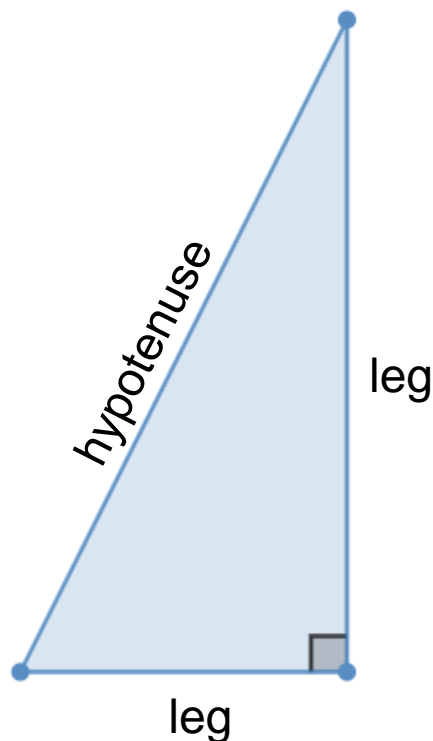
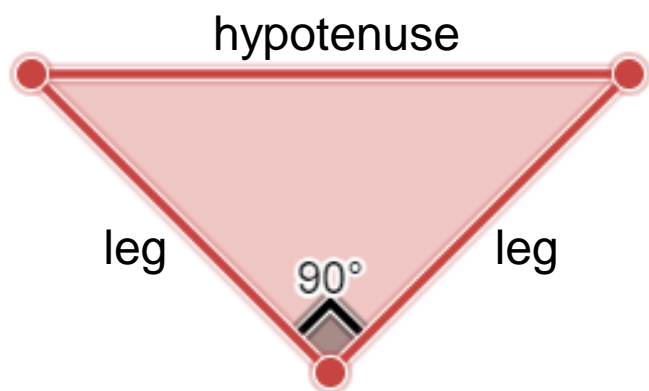
Hierarchy. An arrangement or classification of things according to relative importance or inclusiveness. In geometry, shapes can be classified in a hierarchy based on properties. A hierarchy for quadrilaterals can be seen in the diagram below.



H

Hypotenuse. The longest side of a right triangle. The hypotenuse is always located opposite the right angle in a right triangle.

Examples:



[Imaginary Numbers](#)

[Independently Combined Probability Models](#)

[Integer](#)

[Interpreting Multiplication Expressions](#)

[Interquartile Range](#)

[Interval](#)

[Irrational Numbers](#)

Imaginary Numbers. A number, which when multiplied by itself gives a negative result. An “*i*” after a number means it is an imaginary number. See *also*: [Complex Numbers](#)

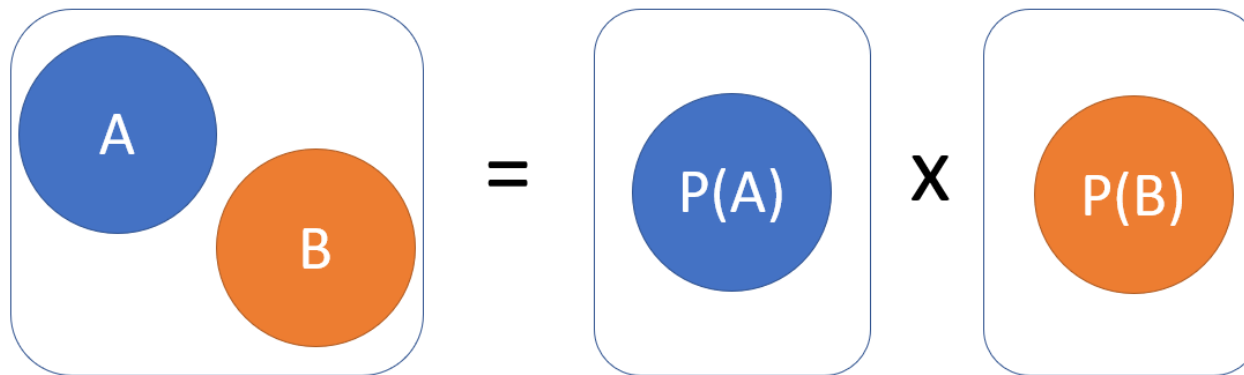
The letter “*i*” is a number, which when multiplied by itself gives -1. So, $i = \sqrt{-1}$.

Normally, we can't find the square root of numbers like -49. But, with imaginary numbers, we can:

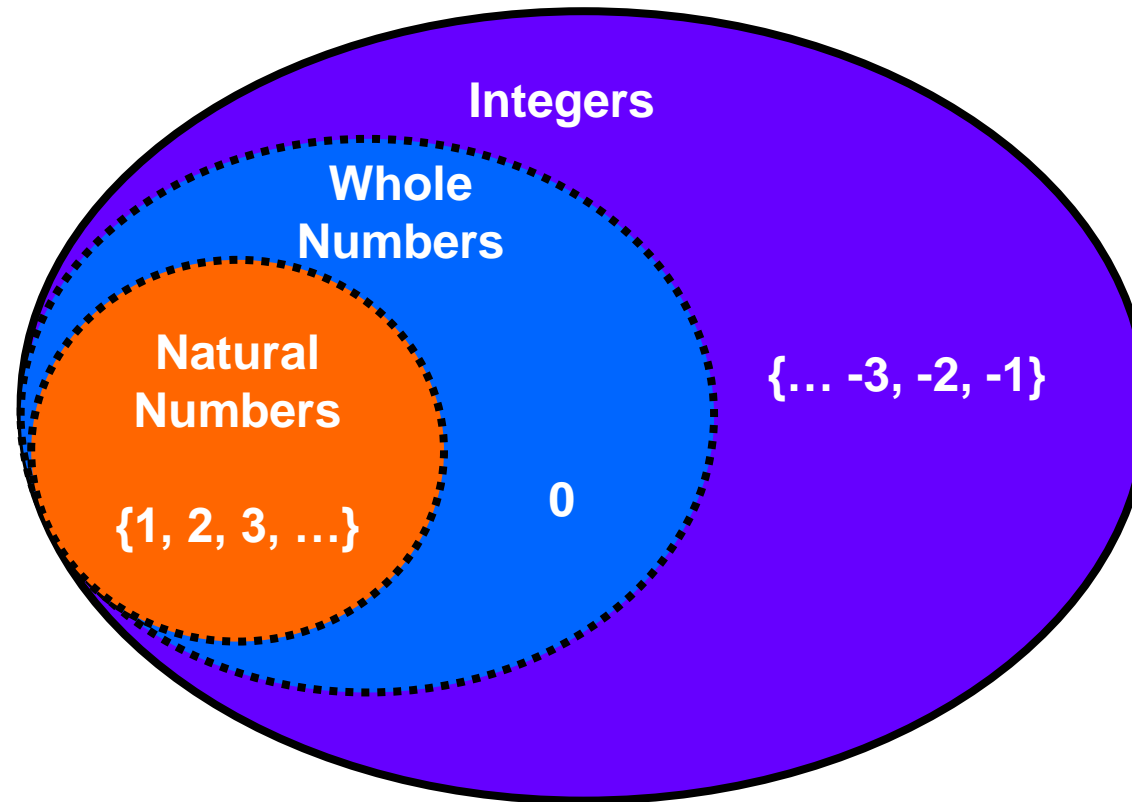
Example: $\sqrt{-49} = \sqrt{49 \cdot -1} = \sqrt{49} \cdot \sqrt{-1} = 7i$

I

Independently Combined Probability Models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.



Integers. The set of whole numbers and their opposites and zero. Also, a number expressible in the form a or $-a$ for some whole number a .



Interpreting Multiplication Expressions. When interpreting multiplication expressions, the factors may be read as ***a*** groups of ***b***, or ***b*** groups of ***a***. For example, 3×6 means how many are in 3 groups of 6 things each: three sixes, or 3×6 means how many are 3 things taken 6 times (6 groups of 3 things each): six threes. The context of the situation will determine which interpretation is required.

Examples:

3×6 as 3 groups of 6 things:

I just bought three six-packs of soda.
How many sodas is that?



3×6 as 3 things taken 6 times:

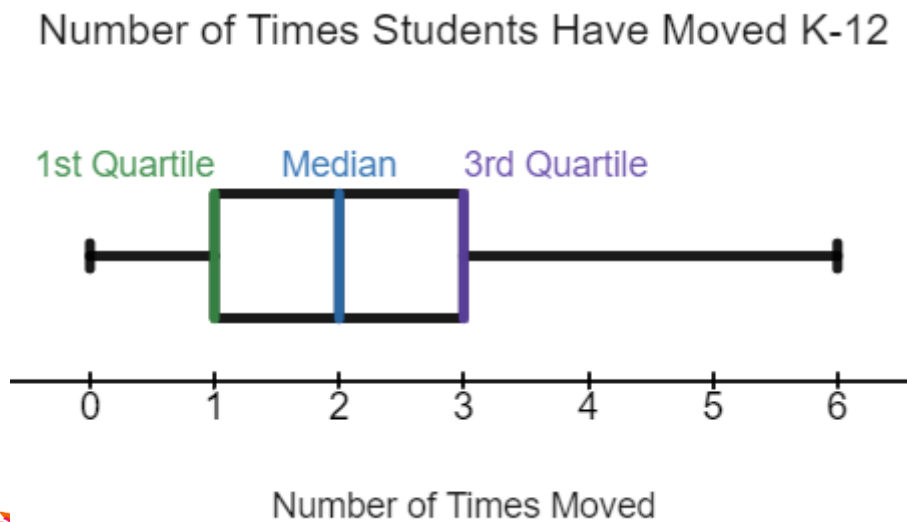
We made three different flyers for the summer carnival. We need six copies of each. How many copies is that?



Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. See *also*: [First Quartile](#), [Third Quartile](#), [Median](#), [Quartile](#).

Example:

In the boxplot below, the interquartile range is $3 - 1 = 2$.



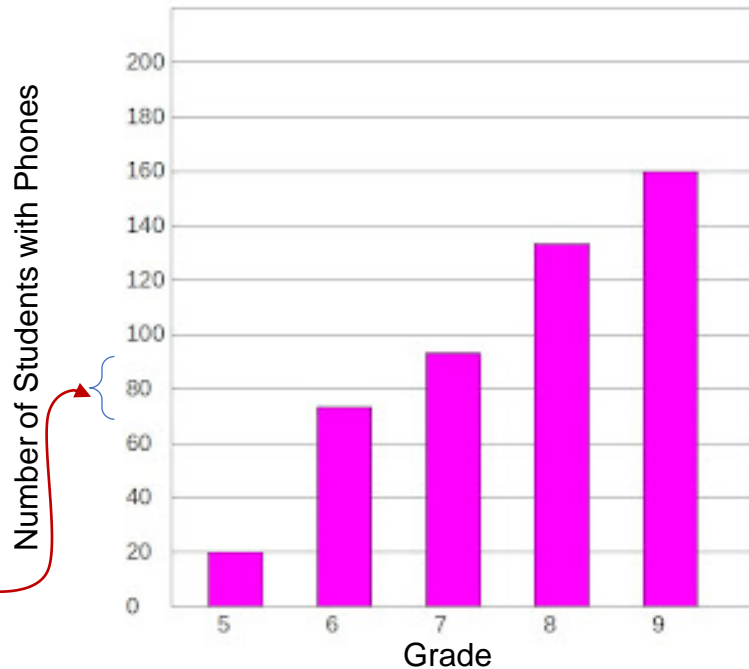
Employee	Age at early retirement
1	56
2	55
3	60
4	51
5	53
6	58
7	56
8	64
9	59
10	42
11	48

Click the image to try an interactive tool using GeoGebra.

1. Type in values for the ages, using whole numbers 1 to 99, inclusive.
2. Show example work steps to find the interquartile range.
3. Type new values for step 1 and observe the changes. To reset the steps (above) click:

Interval. A space or distance between objects, points, or units, especially when making uniform amounts of separation. Also, a set of numbers consisting of all the numbers between a pair of given numbers along with either, both, or neither of the endpoints.

“Students Who Have Cell Phones at School”



The interval is 20

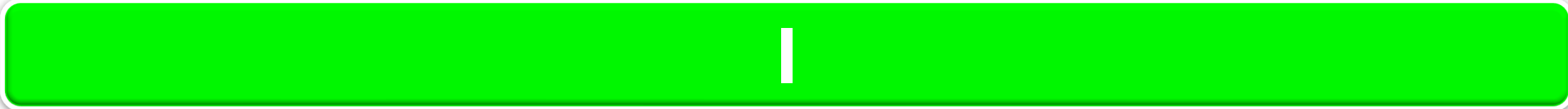
For $a \leq b$, the **closed interval** $[a, b]$ is the set of elements x satisfying $a \leq x \leq b$ (i.e., $a \leq x$ and $x \leq b$). It contains at least the elements a and b . Using the corresponding strict relation " $<$ ", the **open interval** (a, b) is the set of elements x satisfying $a < x < b$ (i.e., $a < x$ and $x < b$).

$$-3 \leq x < 5$$

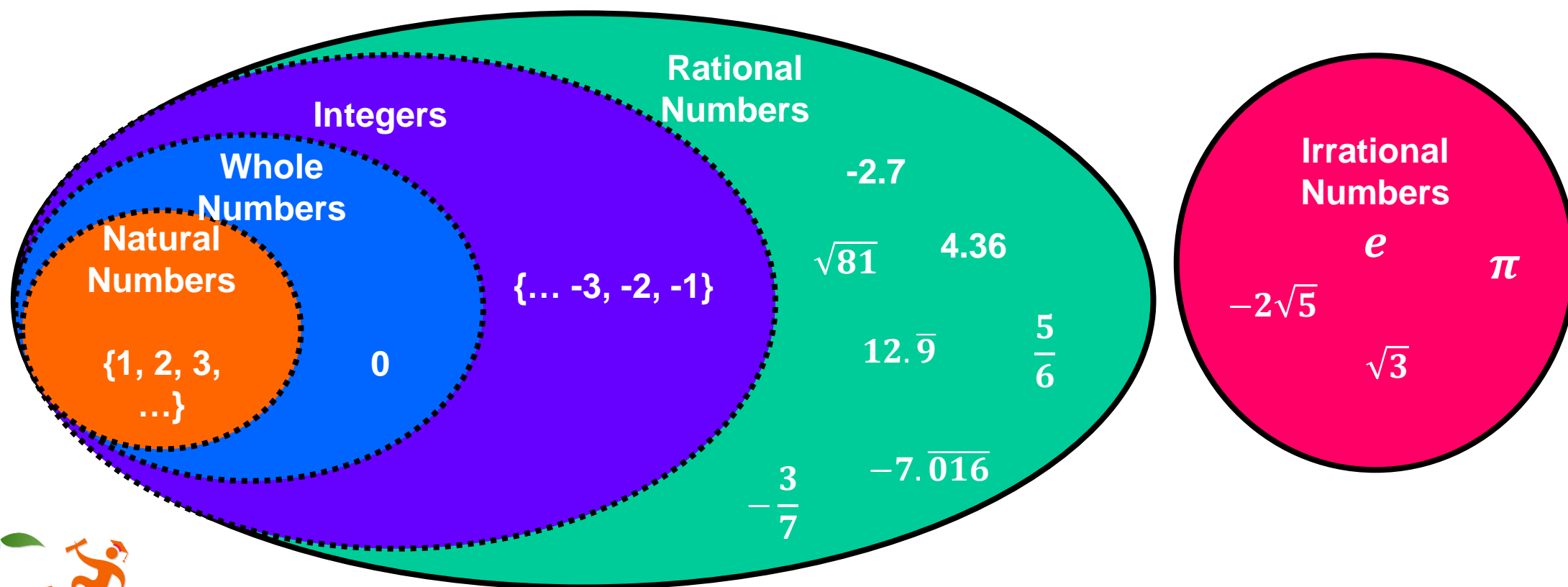
" x is greater than or equal to -3 and less than 5 ."



Interval notation: $[-3, 5)$



Irrational Numbers. A number that cannot be represented as a fraction. A number like pi is irrational because it contains an infinite number of digits with no discernable pattern. In the diagram below, you can see an example of some rational and some irrational numbers.



J

j

[Join Problems](#)

[Joint Discontinuity](#)

[Joint Probability](#)

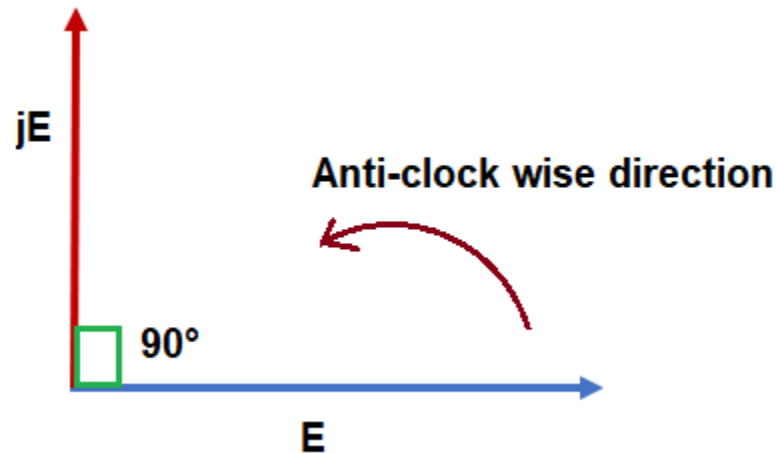
[Joint Variation](#)

[Justify/Justification](#)

J

j. The letter j is used to signify that a number is an imaginary number in electrical engineering. Normally the letter i is used, but in electrical engineering j is used instead to avoid conflict with the symbol for current. See also: [Imaginary Numbers](#), [Vector](#)

j Operator is a mathematical operator which when multiplied with any vector, rotates that vector by 90 degree in anti-clockwise direction.



J

Join Problems. Join problems involve a direct or implied action in which a set is increased by a particular amount. Join problems can be categorized into three structures: result unknown, change unknown, and start unknown. See also: [Problem Types](#)

Examples:

Result Unknown

Robyn had 5 toy cars. Her parents gave her 2 more toy cars for her birthday. How many toy cars did she have then?

Change Unknown

Robyn had 5 toy cars. Her parents gave her some more toy cars for her birthday. Then she had 7 toy cars. How many toy cars did her parents give her for her birthday?

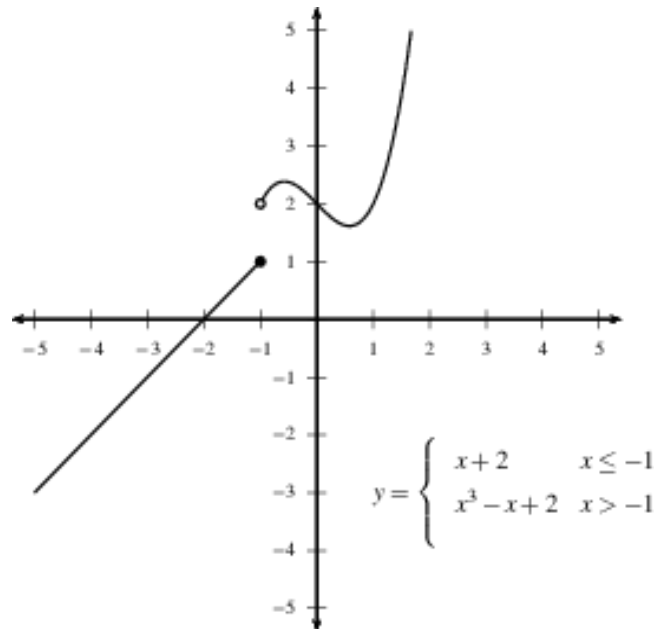
Start Unknown

Robyn had some toy cars. Her parents gave her two more toy cars for her birthday. Then she had 7 toy cars. How many toy cars did Robyn have to before her birthday?

J

Joint Discontinuity. Joint discontinuities are nonremovable and occur when a function has two ends that do not meet even if the hole is filled in. In order to satisfy the vertical line test and make sure the graph is truly that of a function, only one of the end points may be filled. Below is an example of a function with a jump discontinuity. For joint discontinuity, $\lim_{x \rightarrow a^+} f(x)$ and $\lim_{x \rightarrow a^-} f(x)$ both exist but are different values.

Example:



J

Joint Probability. The probability that two events will both occur.

Example: What is the probability of rolling two threes on a single toss of two fair dice.

Number of possible outcomes when a dice is rolled = 6 {1, 2, 3, 4, 5, 6}

Let A be the event of rolling a 3 on the first die and B be the event of rolling a 3 on the second die.

The probability of a three occurring on each die is $\frac{1}{6}$.

$$\begin{aligned}P(A) &= \frac{1}{6} \\P(B) &= \frac{1}{6} \\P(A \cap B) &= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}\end{aligned}$$

J

Joint Variation. A variable varies directly as the product of two or more variables.

Joint Variation

- $y = kxz$ for some nonzero constant, k
- $k = \frac{xz}{y}$
- In this case, y varies jointly with x and z .
- y is jointly proportional to x and z .

J

Justification (n). An explanation, definition, postulate, or theorem which enables a conclusion to be drawn. **Justify (v).** To provide an explanation, definition, postulate, or theorem which enables a conclusion to be drawn.



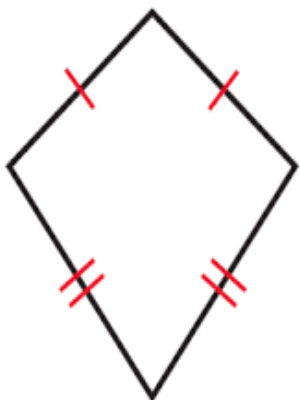
K

Kite

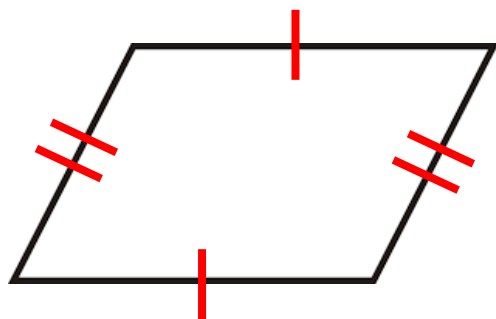
K

Kite. A quadrilateral with two pairs of equal-length sides that are adjacent to each other. Parallelograms also have two pairs of equal-length sides. However, in a parallelogram, these equal-length sides are opposite one another. See *also*: [Polygon](#), [Quadrilateral](#).

Examples:

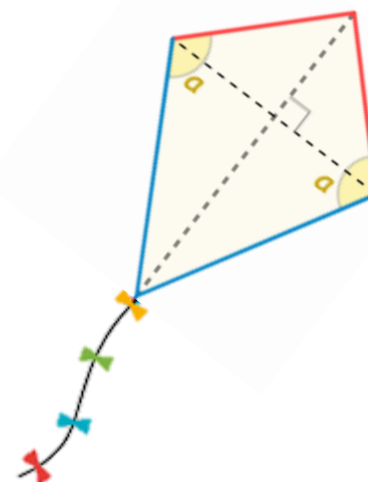


Kite



Parallelogram

In the example image of a kite shown below, the dashed lines are diagonals, which meet at a right angle and one of the diagonals bisects (cuts equally in half) the other. The angles identified as **a** are congruent where the pairs meet, as shown below.



[Back to Start](#)

L

[Line](#)

[Line Segment](#)

[Linear Function](#)

[Logarithm](#)

[Logarithmic Function](#)

[Lower Extreme](#)

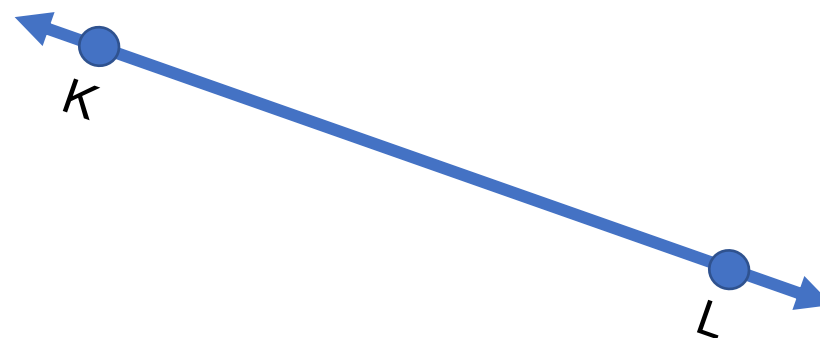
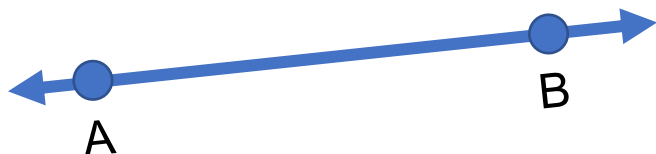


[Back to Start](#)

L

Line. A line is a one-dimensional figure, which has length but no width. A line is made of a set of points which is extended in opposite directions infinitely with zero curvature.

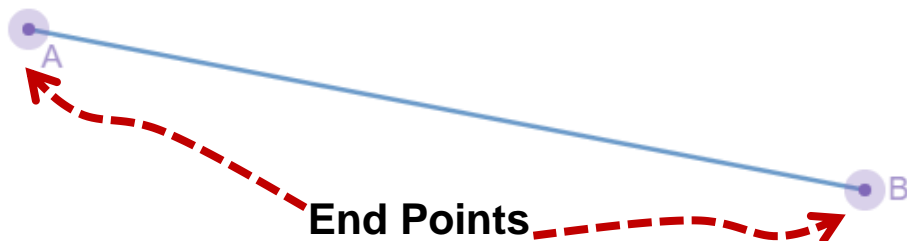
Each of the images below represent lines.



L

Line Segment. A line segment is part of a line which has defined endpoints.

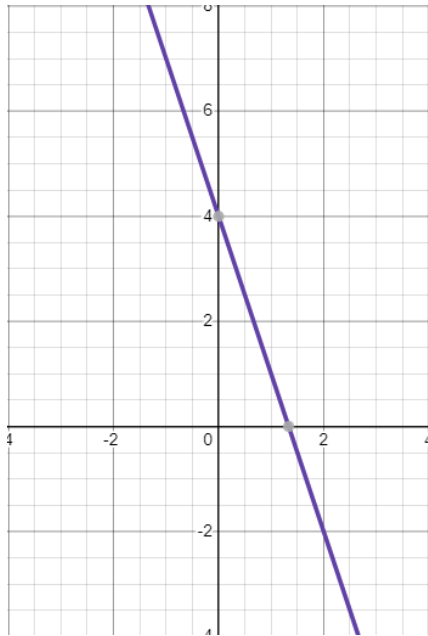
Each of the images below represent line segments.



L

Linear Function. A function whose graph is a straight line. A linear function describes a gradual rate of change, either positive or negative. When drawn, it presents a straight line.

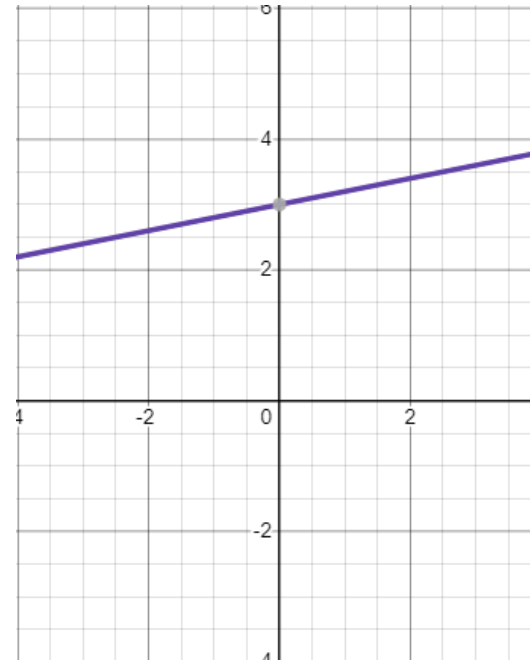
Examples:



$$y = -3x + 4$$

$$y - 7 = -3(x - (-1))$$

$$-3x - y + 4 = 0$$



$$y = \frac{1}{5}x + 3$$

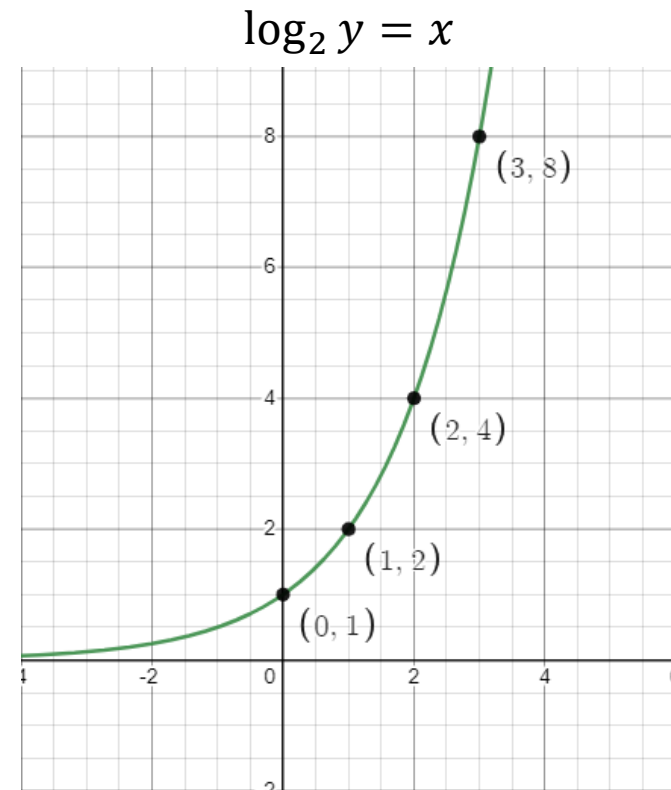
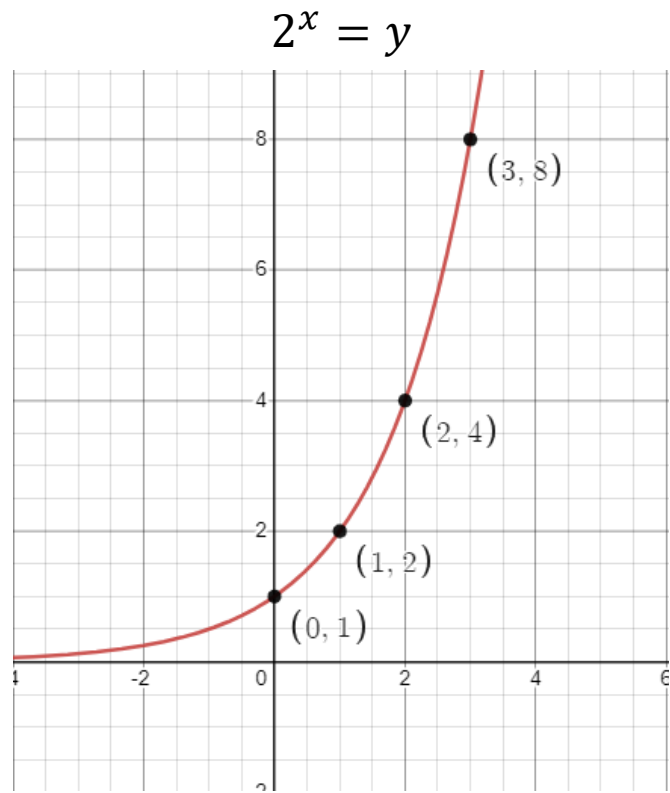
$$y - 4 = \frac{1}{5}(x - 5)$$

$$\frac{1}{5}x - y + 3 = 0$$

L

Logarithmic Function. The inverse of an exponential function.

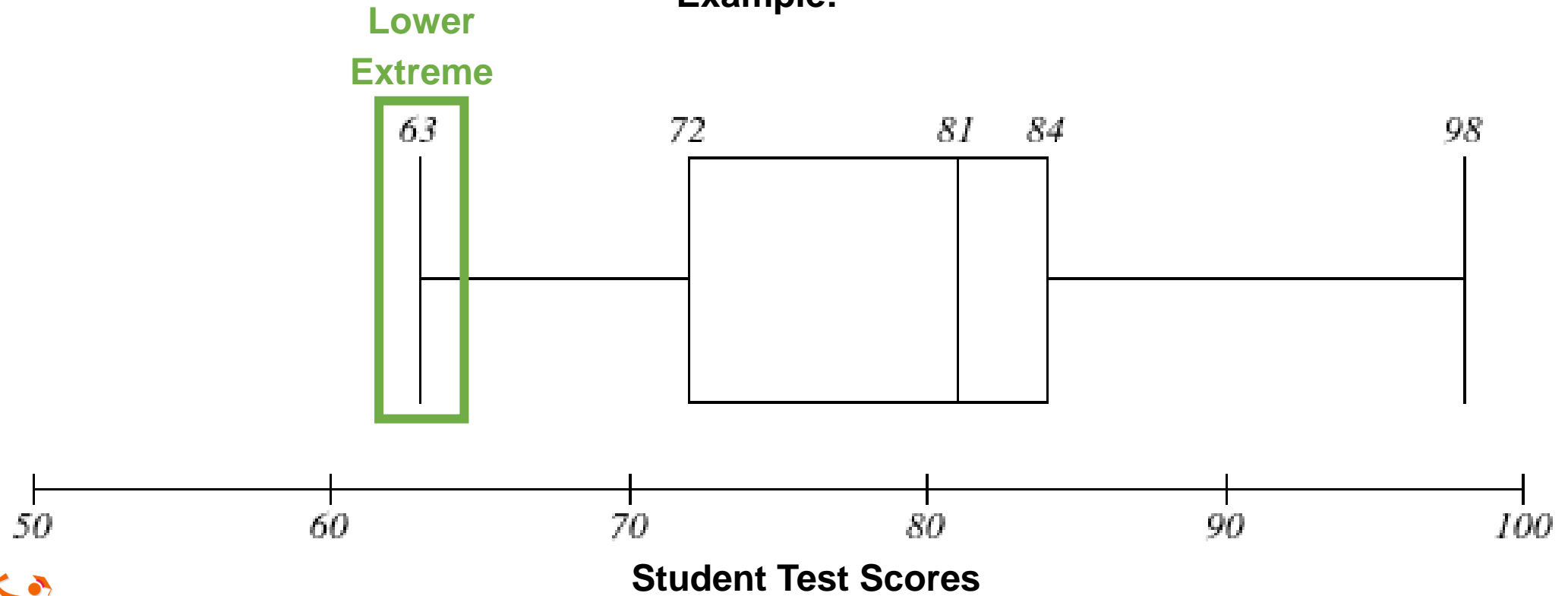
Examples:



L

Lower Extreme. The smallest value in the data set; also called the minimum. See *also*: [Upper Extreme](#)

Example:



M

[Mathematical Proficiency](#)

[Mathematize](#)

[Mean](#)

[Mean Absolute Deviation](#)

[Median](#)

[Memorization vs Memory](#)

[Midline](#)

[Midsegment](#)

[Mindset](#)

[Model with Mathematics](#)

[Monomial](#)

[Multiple](#)

[Multiplication and Division within 100](#)

[Multiplicative Comparison](#)

[Multiplicative Inverses](#)

M

Mathematical Proficiency. Conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, productive disposition (National Research Council, 2001).

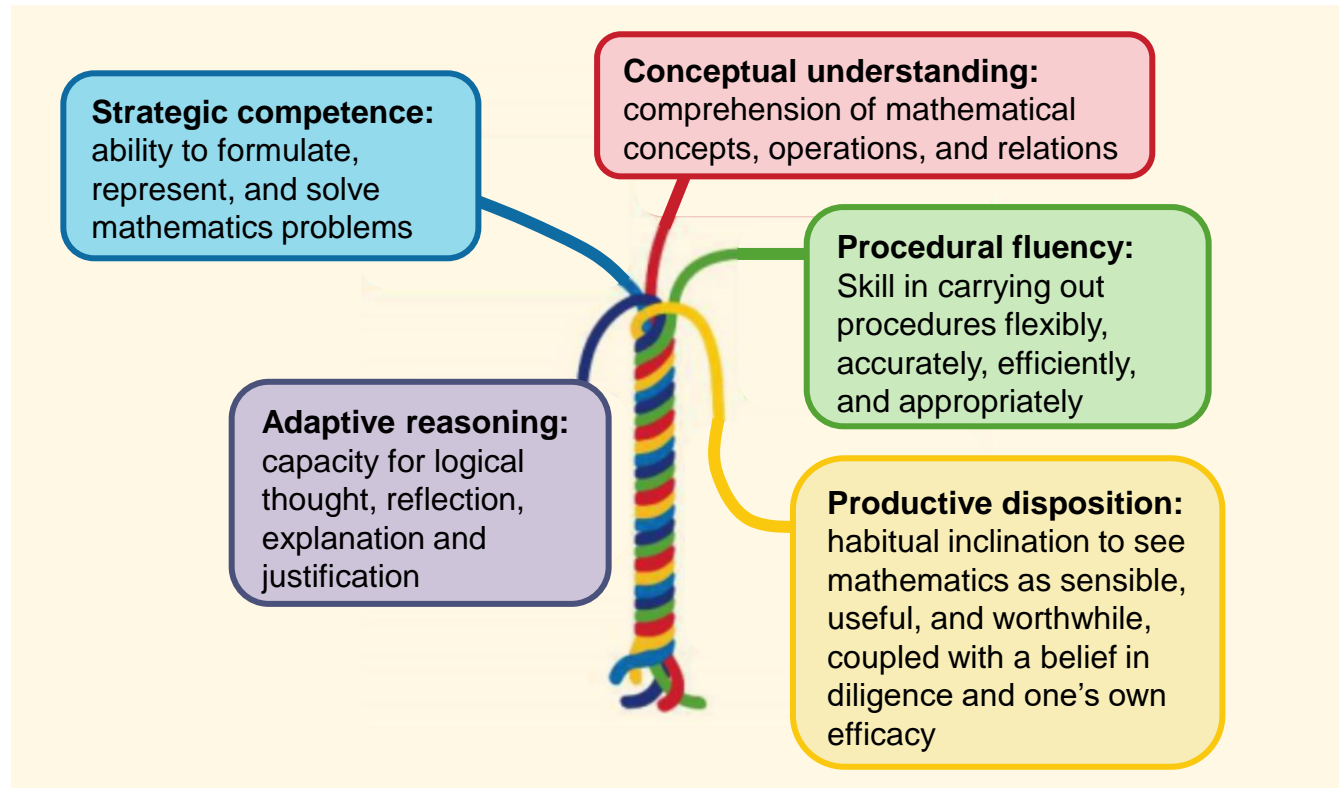


Image adapted from: National Research Council. (2001).
Adding It Up: Helping Children Learn Mathematics

M

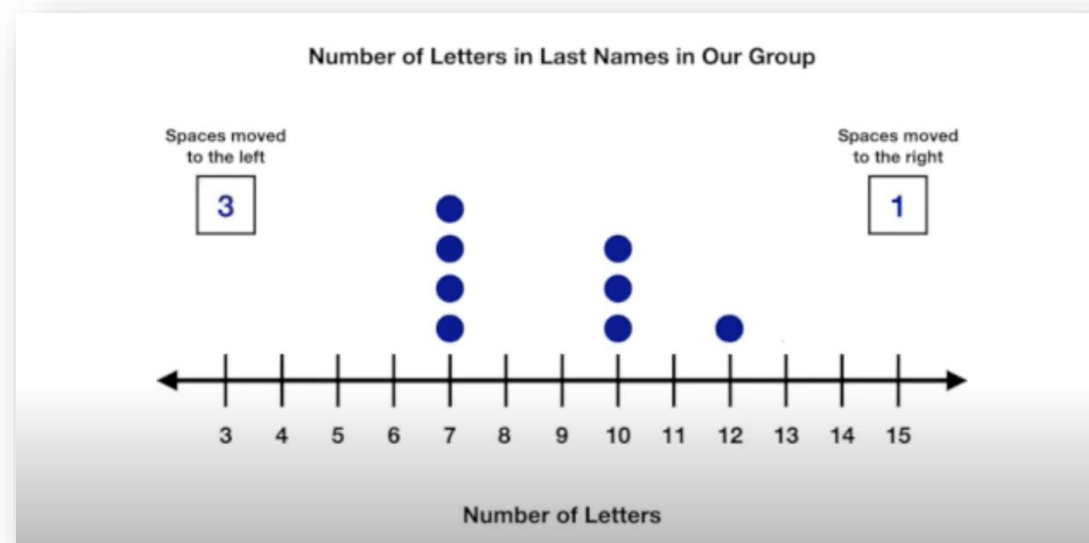
Mathematize. To see, experience, and draw out the mathematics in a situation.

See also: [Model with Mathematics](#).



M

Mean. A measure of center in a set of numerical data. The mean for a set of data can be found, conceptually, using a dot plot of the data. Each data point can be moved toward the center to find the “balance point.” After building a conceptual understanding of the mean, students can transfer this understanding to compute the mean by adding the data values and then dividing by the number of data values in the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21. (To be more precise, this defines the *arithmetic mean*.)

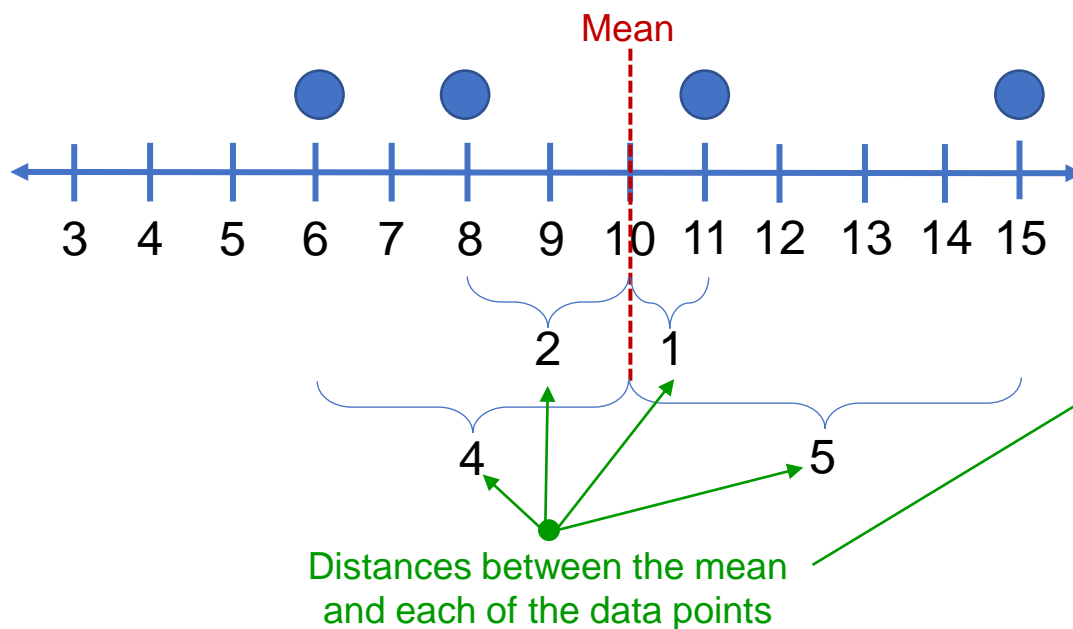


Click the image to watch a brief video showing the mean as a balance point.

M

Mean Absolute Deviation. A measure of the average absolute distance between each data value and the mean of a data set. Mean absolute deviation is a parameter or statistic that measures the spread, or variation, in your data.

To find the mean absolute deviation, we measure the absolute (value) distance between each data point and the mean. Then, find the average (mean) of those distances.



Find the mean of these distances to find the Mean Absolute Deviation.

M

Median. A measure of center in a set of numerical data. The median of a list of data points is the value appearing at the center of a sorted version of the list. If the list contains an even number of values, the mean of the two central values is the median. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Trae Young, Atlanta Hawks Point Guard, Scored the following number of points in each game of the 2022 post-season:

8, 9, 11, 24, 24, 25, 38

The median is the middle value.

Median



Points Scored by Trae Young During the 2022 Post-Season

M

Memorization vs Memory. Rote memorization is void of strategy or meaning whereas learning from memory relies on strategy and/or is connected to other mathematical ideas through reasoning or context or both.

I don't remember...
9...18...27...
...?



$$9 \times 7 = ?$$

I don't remember.
But I know $10 \times 7 = 70$ and that's
one 7 too many.
So, 63!

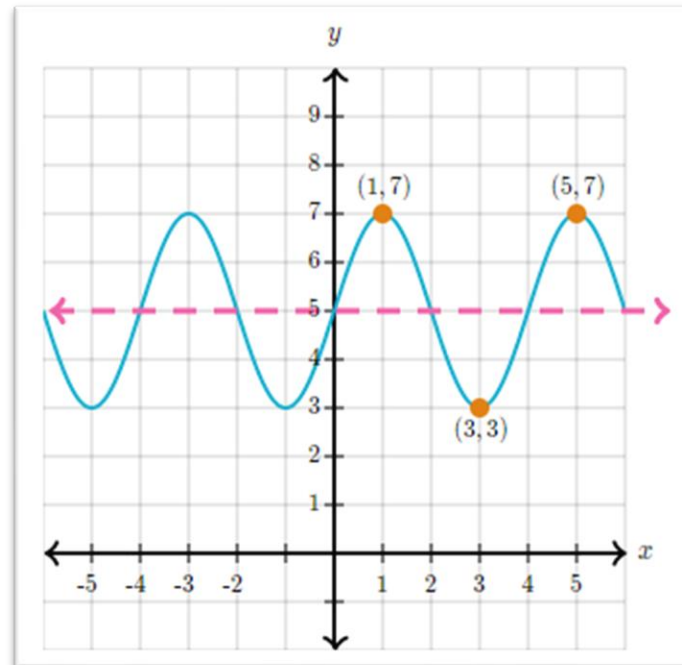


M

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Example:

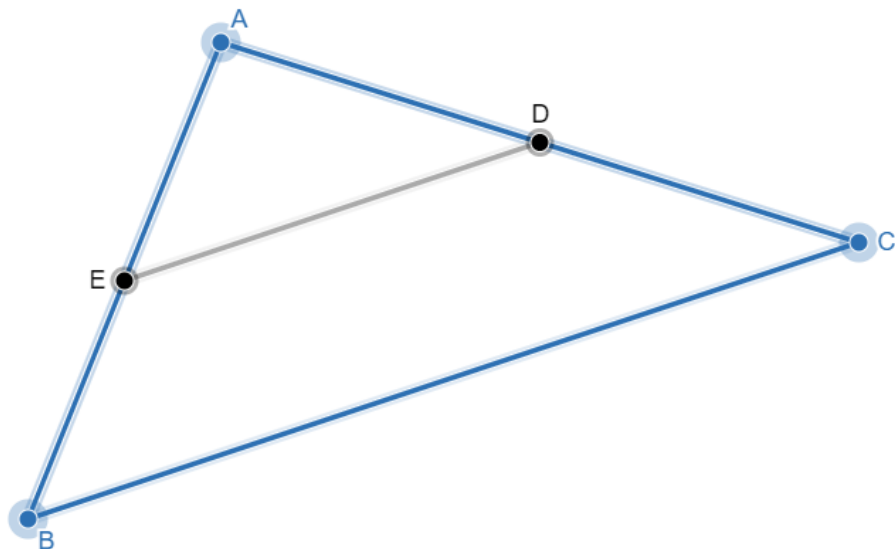
The horizontal line that passes exactly between $y = 7$ (the maximum value) and $y = 3$ (the minimum value) is $y = 5$, so that's the **midline**



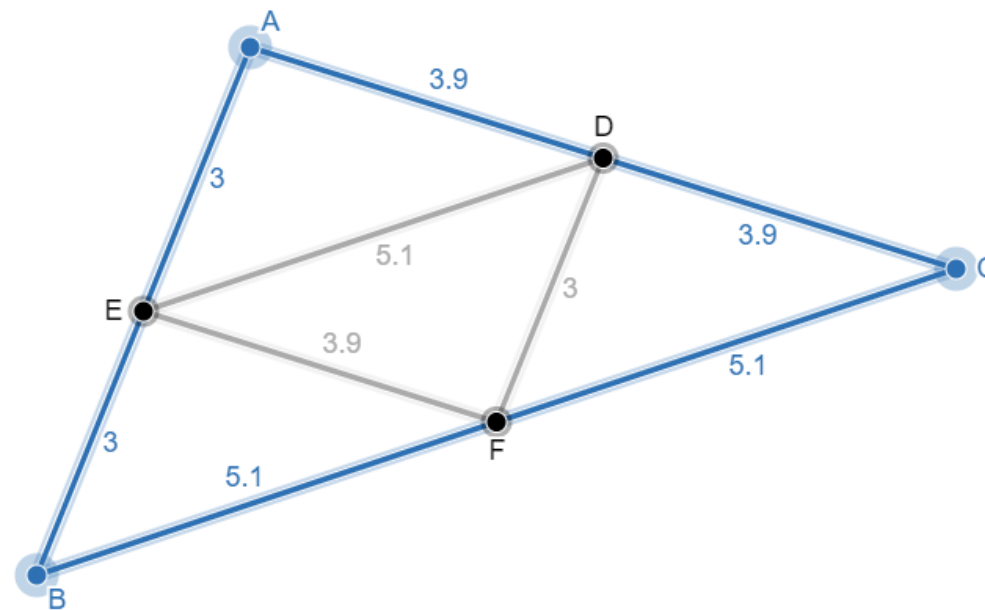
M

Midsegment. A line segment that joins the midpoints or centers of two adjacent sides of a triangle.

In the figure below, D is the midpoint of AC and E is a midpoint of AB. DE is a midsegment of triangle ABC.

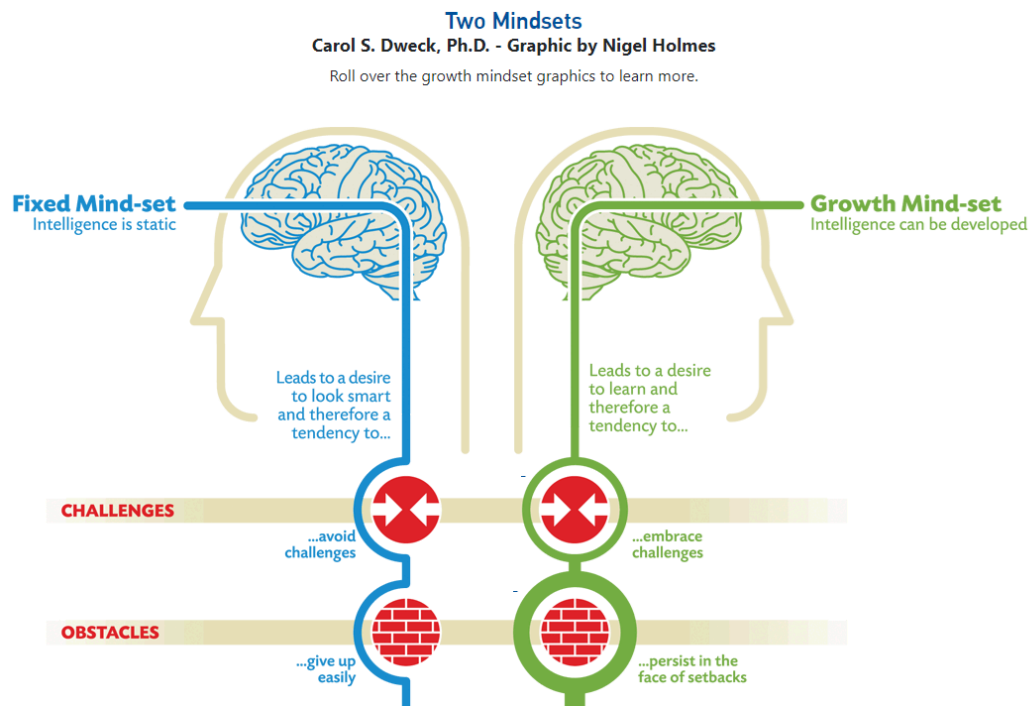


In the figure below, all midpoints are connected, showing all of the midsegments (and their lengths) of triangle ABC. What do you notice?



M

Mindset. A set of beliefs that shape how you make sense of the world and yourself. It influences how you think, feel, and behave in any given situation. there are two basic mindsets: fixed and growth. If you have a fixed mindset, you believe your abilities are fixed traits and therefore can't be changed. if you have a growth mindset, you believe that your talents and abilities can be developed over time through effort and persistence.



**Click the image
for more
information
about mindsets.**

M

Model with Mathematics. Move explicitly between real-world scenarios and mathematical representations of these scenarios.



Example:

Innovative roller coaster engineers set out to design a thrilling roller coaster, but what makes a roller coaster exciting and fun?

Create a mathematical model that ranks the roller coasters in Georgia according to a thrill factor that **you** define.

M

Monomial. An algebraic expression consisting of one term. Some examples of monomials can be found below.

$$3x^2y$$

$$73n$$

$$m$$

$$8abc$$

$$-s$$

M

Multiple. A product that is created when one number is multiplied by another number.

Examples:

Number	First Ten Multiples (Non-Zero)
3	3, 6, 9, 12, 15, 18, 21, 24, 27, 30,...
6	6, 12, 18, 24, 30, 36, 42, 48, 54, 60,...
10	10, 20, 30, 40, 50, 60, 70, 80, 90, 100,...
12	12, 24, 36, 48, 60, 72, 84, 96, 108, 120,...
25	25, 50, 75, 100, 125, 150, 175, 200, 225, 250,...

M

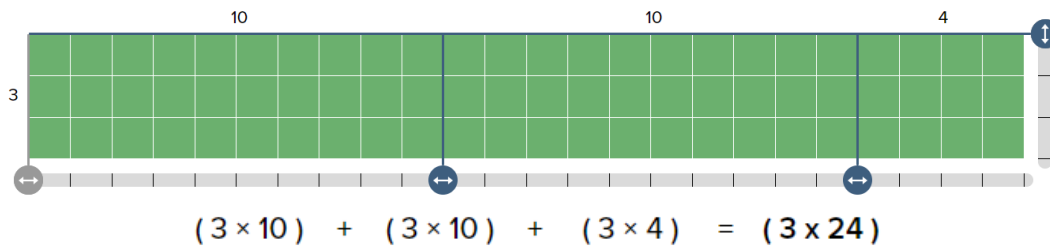
Multiplication and Division within 100. Multiplication or division of two whole numbers with whole number answers, and with a product or dividend in the range 0-100.

Example: A box of chocolates has three layers. Each layer has 24 chocolates. How many chocolates are in the box?

Multiplication within 100

$$24 \times 3 = 72$$

Bold values must be within the range 0-100



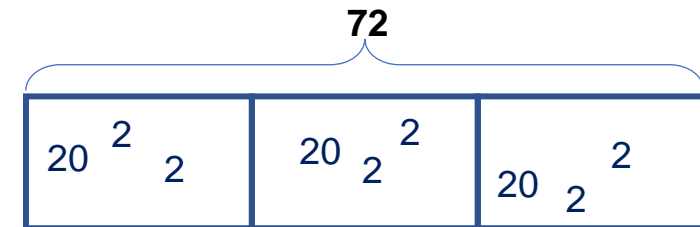
$$30 + 30 + 12 = 72$$

Example: A box of chocolates has 72 pieces. There are three layers of chocolates with the same number of pieces in each layer. How many chocolates are in each layer?

Division within 100

$$72 \div 3 = 24$$

Bold values must be within the range 0-100



[Back to Start](#)

M

Multiplicative Comparison. Multiplicative comparisons focus on comparing two quantities by showing that one quantity is a specified number of times larger or smaller than the other.

Example: *Deb ran 3 miles. Karen ran 5 times as many miles as Deb.
How many miles did Karen run?*

Deb

3 miles

Karen

3 miles

3 miles

3 miles

3 miles

3 miles

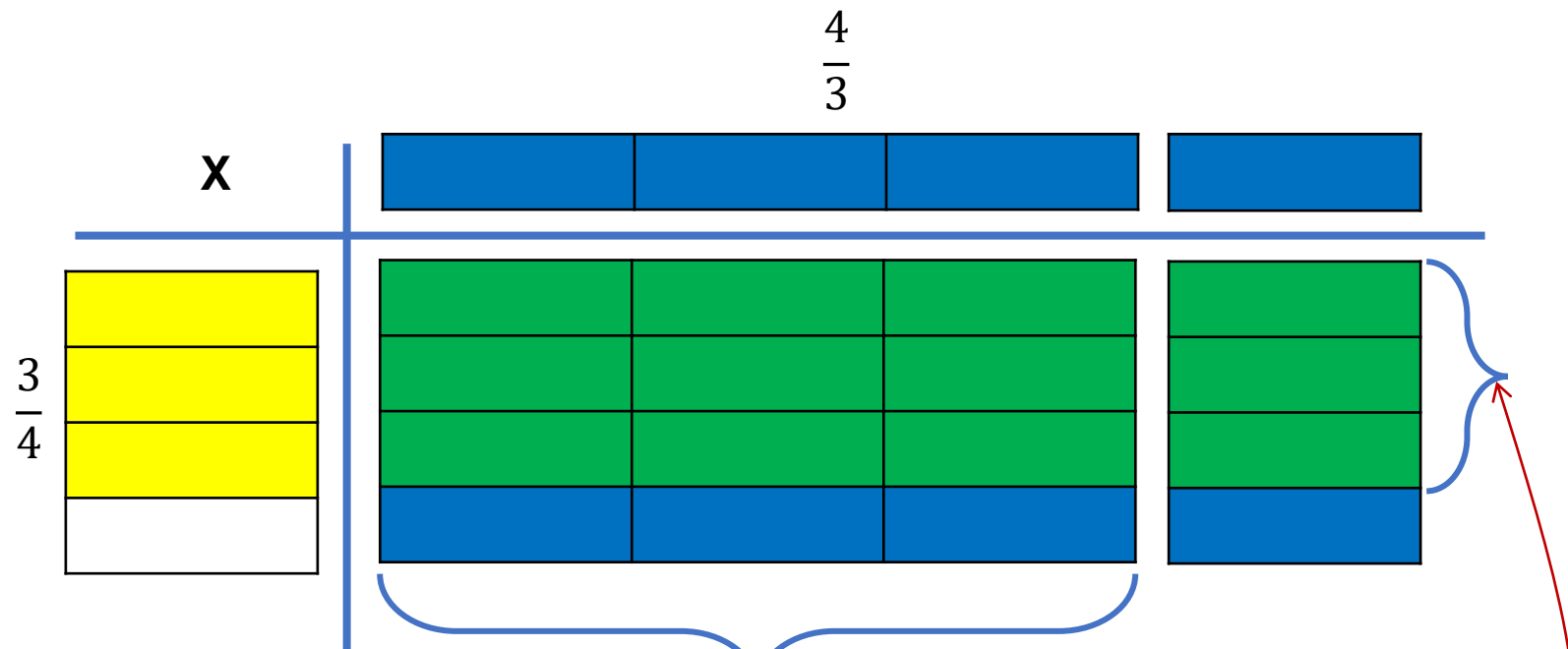
5 groups of 3 miles is the same as $5 \times 3 = 15$ miles.
Karen ran 15 miles.

M

Multiplicative Inverses. Two numbers whose product is 1 are multiplicative inverses of one another.

Example: $\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverses of one another because

$$\frac{3}{4} \times \frac{4}{3} = \frac{4}{3} \times \frac{3}{4} = 1$$



The whole consists of 12 equal pieces. Only 9 are green here. But there are three more here, making twelve twelfths or 1 whole.

N

[Net](#)

[Number](#)

[Number Line Diagram](#)

[Numeracy](#)

[Numeral](#)

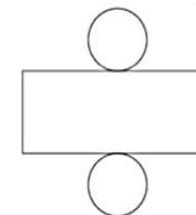
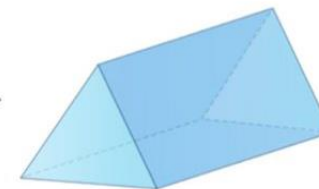
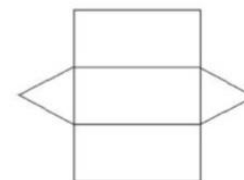
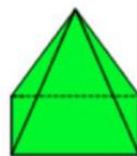
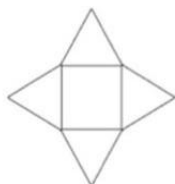
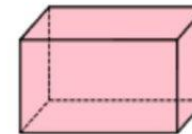
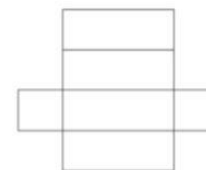
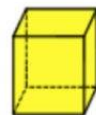
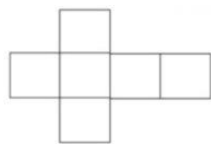
[Numerator](#)



[Back to Start](#)

N

Net. A plane diagram of a solid figure.



Click an image to explore nets of solid figures using GeoGebra.

N

Number. A count of a quantity or measurement of units. A number can be represented in many ways such as a numeral, a drawing, a word, or a picture.

Example:

In what ways can the number of cookies on the plate be shown?

Picture Arrangement:



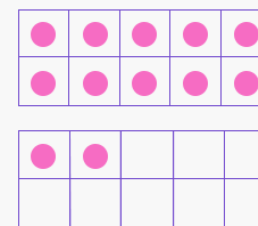
Numeral: 12

The digits one and two can be used to write this quantity (number) as a numeral.

Word Form:

There are twelve cookies on the plate.

12
Twelve



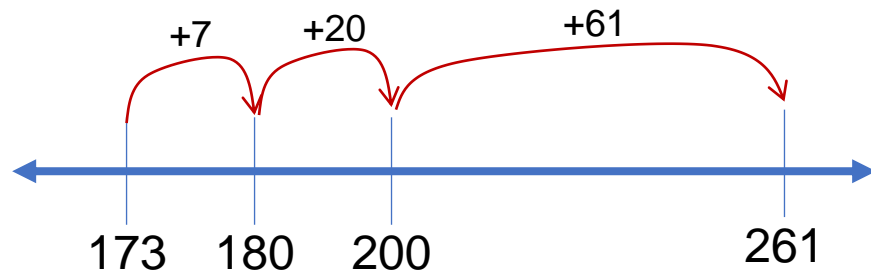
[Back to Start](#)

N

Number Line Diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Example:

Eddie and Kenny played a game. At the end of the game, Eddie had scored 261 points and Kenny scored 173 points. How many more points did Eddie score than Kenny?

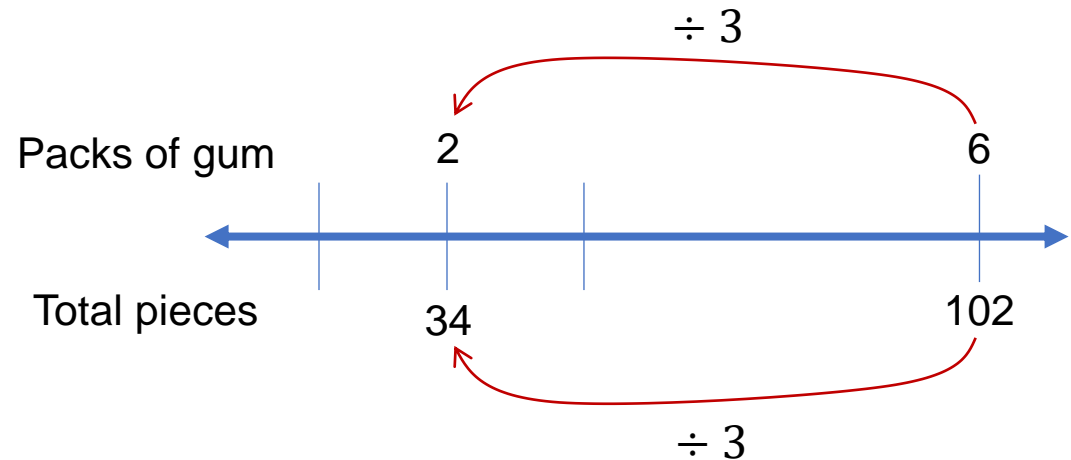


$$61 + 20 + 7 = 87$$

Eddie scored 87 more points than Kenny.

Example:

In 6 packs of gum there are 102 pieces. How many pieces of gum would be in 2 packs of gum?

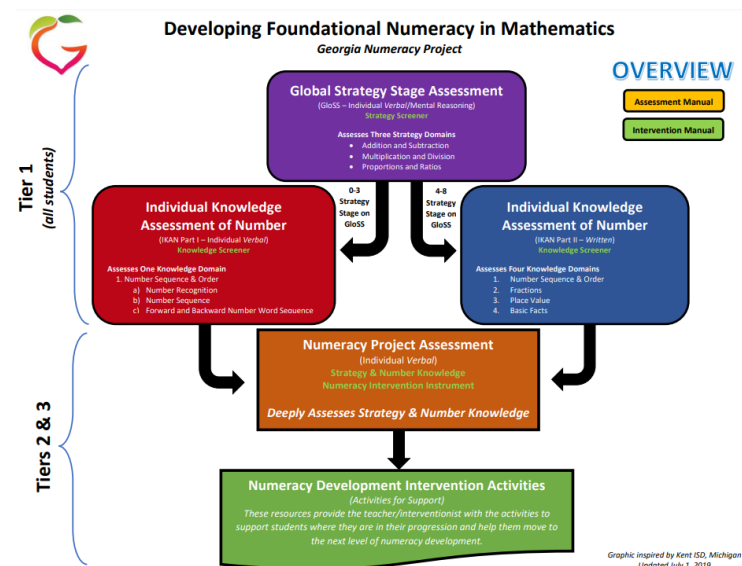
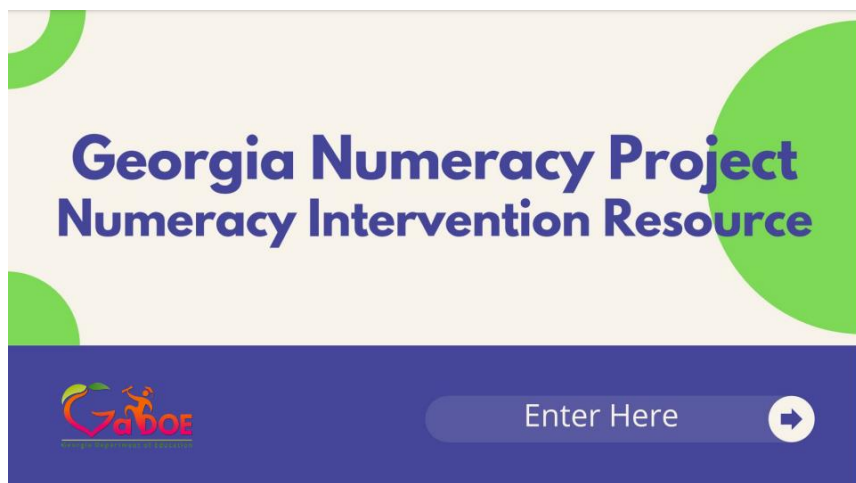


[Back to Start](#)

N

Numeracy. The ability to use mathematical ideas involving number and quantity efficiently, flexibly, and accurately to make sense of the world and solve problems.

Click either of the images below to learn more about how to help students develop a solid foundation in numeracy.



N

Numeral. A symbol or name used alone or in a group to represent a number. See *also*: [Digit](#), [Number](#).

Example:

12 is a symbol used to represent a quantity or number of objects. **12** is composed of two digits: 1 and 2)



N

Numerator. In fraction notation, the numerator shows how many of the parts indicated by the denominator are counted. See *also*, [Denominator](#), [Fraction](#)

Example: In the image below, there are 12 eggs in the whole carton, so each egg represents $\frac{1}{12}$ of the whole carton of eggs. $\frac{4}{12}$ of the eggs in the carton are brown. The numerator, 4, indicates this count of four eggs (each of which represent $\frac{1}{12}$ of the whole) are brown.



0

[Obtuse Angle](#)

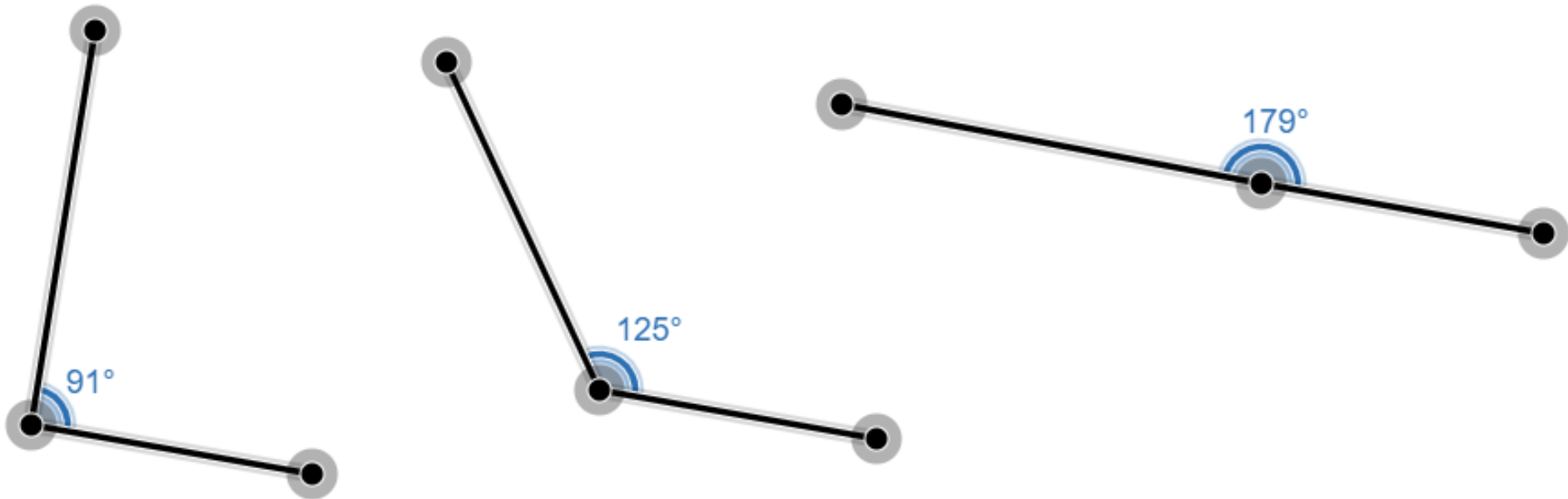
[Octagon](#)

[Opposite](#)

O

Obtuse Angle. An angle measuring between 90° and 180° . A few examples of some obtuse angles are shown below. See *also*: [Angle](#), [Acute Angle](#), [Right angle](#).

Examples:



O

Octagon. In geometry, an octagon is an eight-sided polygon or 8-gon. See *also*: [Polygon](#)

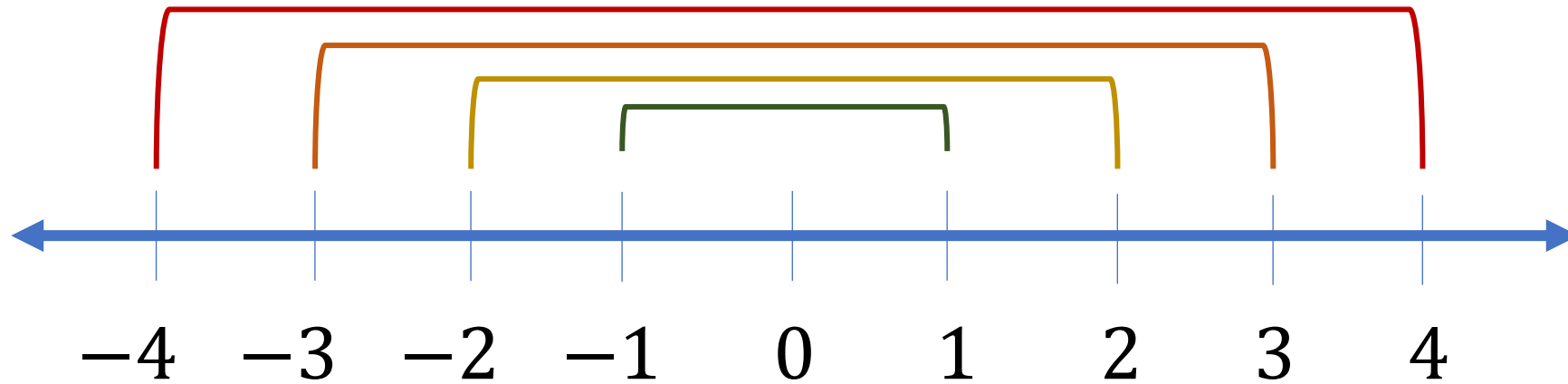
Examples:



0

Opposite. A number with the same absolute value as a given number, but with a different sign.

Opposite numbers are equidistant from zero on a number line and their sum is always zero. Example: 3 and -3 are opposites.



P

[Parallel](#)

[Parallelogram](#)

[Part-Part-Whole Problems](#)

[Pentagon](#)

[Percent Rate of Change](#)

[Perpendicular](#)

[Perseverance](#)

[Pi](#)

[Place, Value, and Place Value](#)

[Polygon](#)

[Polyhedron](#)

[Polynomial](#)

[Polynomial Expression](#)

[Prism](#)

[Probability](#)

[Probability Distribution](#)

[Probability Model](#)

[Problem Types](#)

[Procedural Understanding](#)

[Procedure](#)

[Properties of Operations](#)

[Properties of Equality](#)

[Properties of Inequality](#)

[Proportional Relationship](#)

[Pythagorean Theorem](#)

P

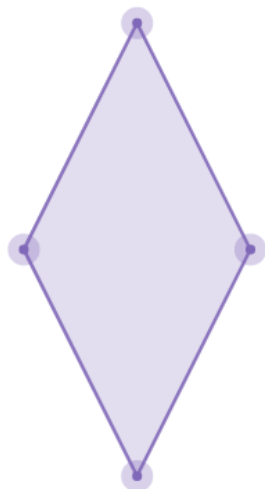
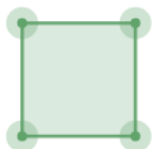
Parallel. Lines are parallel if they lie in the same plane and are the same distance apart over their entire length.



P

Parallelogram. A quadrilateral with both pairs of opposite sides parallel. See *also*: [Polygon](#), [Quadrilateral](#).

Click one of the parallelograms below to explore parallelograms using a dynamic Desmos geometry tool.



P

Part-Part-Whole Problems. Part-Part-Whole problems involve static relationships between a particular set and its two disjoint subsets. There is no direct or indirect action in these types of problems and there is no change over time. There are three categories of part-part-whole problems. See also: [Problem Types](#)

Whole Unknown

8 brown birds and 6 black birds were sitting in a tree. How many birds were sitting in the tree?

Part Unknown

14 birds were sitting in a tree. 6 of the birds were brown birds and the rest were black birds. How many black birds were sitting in the tree?

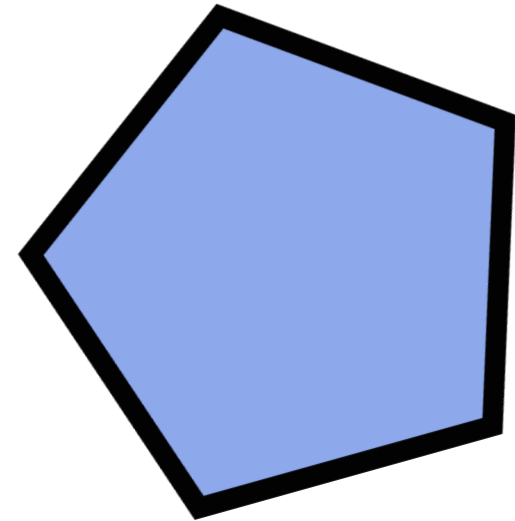
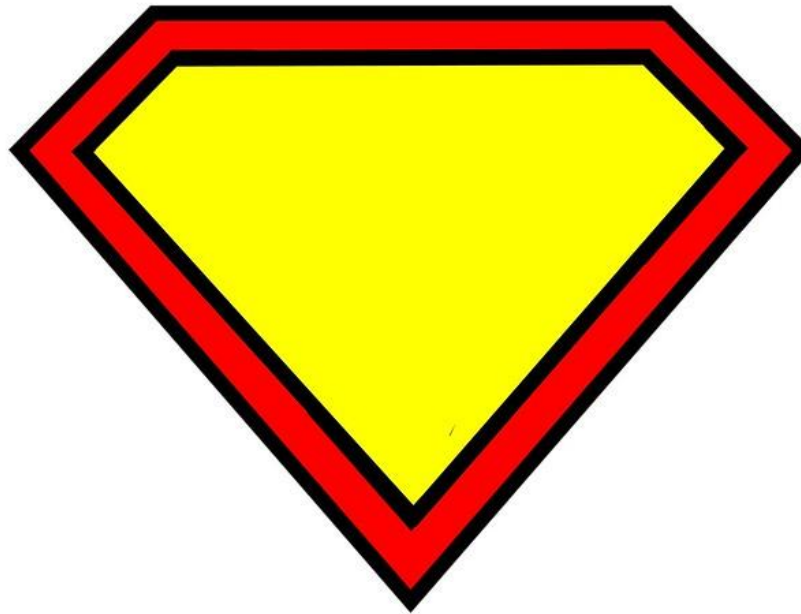
Both Parts Unknown

14 birds were sitting in a tree. Some of the birds were brown birds and the some were black birds. How many black birds and brown birds could be sitting in the tree?

P

Pentagon. In geometry, a pentagon is a five-sided polygon or 5-gon. See *also*: [Polygon](#)

Examples:



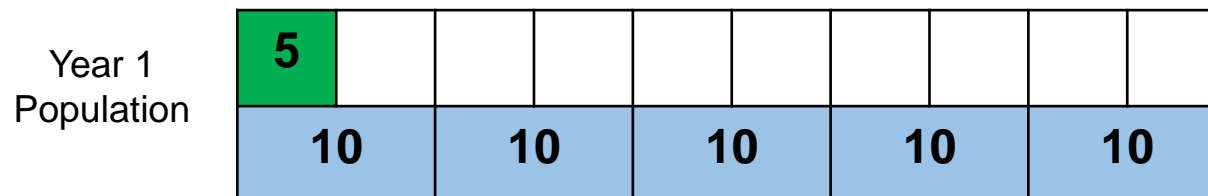
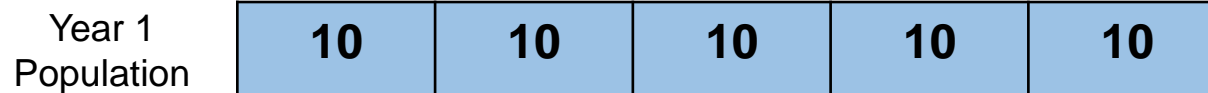
P

Percent Rate of Change. A rate of change expressed as a percent.

Example:

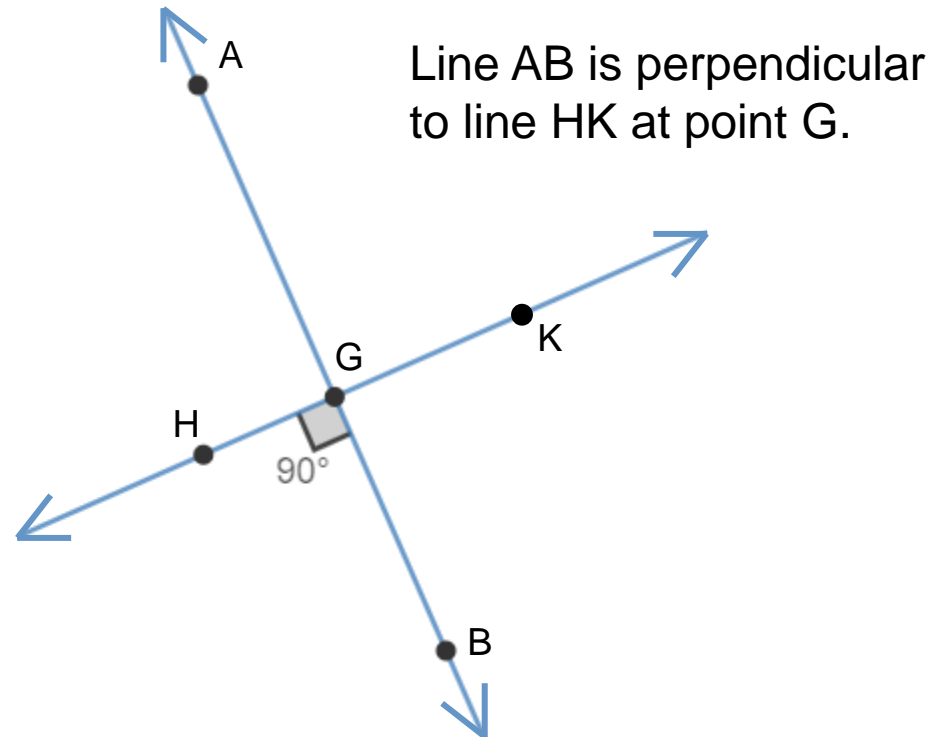
If a population grows from 50 to 55 in a year, find the percent rate of change in population from year 1 to year 2.

We can find the percent rate of change by finding what part of the original 50 the growth of 5 is. The growth of 5 is $\frac{1}{10}$ of the original population of 50. So, the rate of change is $\frac{1}{10} = 10\%$



P

Perpendicular. In elementary geometry, two geometric objects are perpendicular if they intersect at a right angle. A line is said to be perpendicular to another line if the two lines intersect at a right angle.



P

Perseverance. Continued effort to do or achieve something despite difficulties, failure, or opposition.

“It always seems impossible until it's done.”
— **Nelson Mandela**

“I’m convinced that about half of what separates the successful entrepreneurs from the non-successful ones is pure perseverance.”
– **Steve Jobs**

Excellence is perseverance in the presence of obstacles.

“If you fell down yesterday,
stand up today.”
— **H.G. Wells**

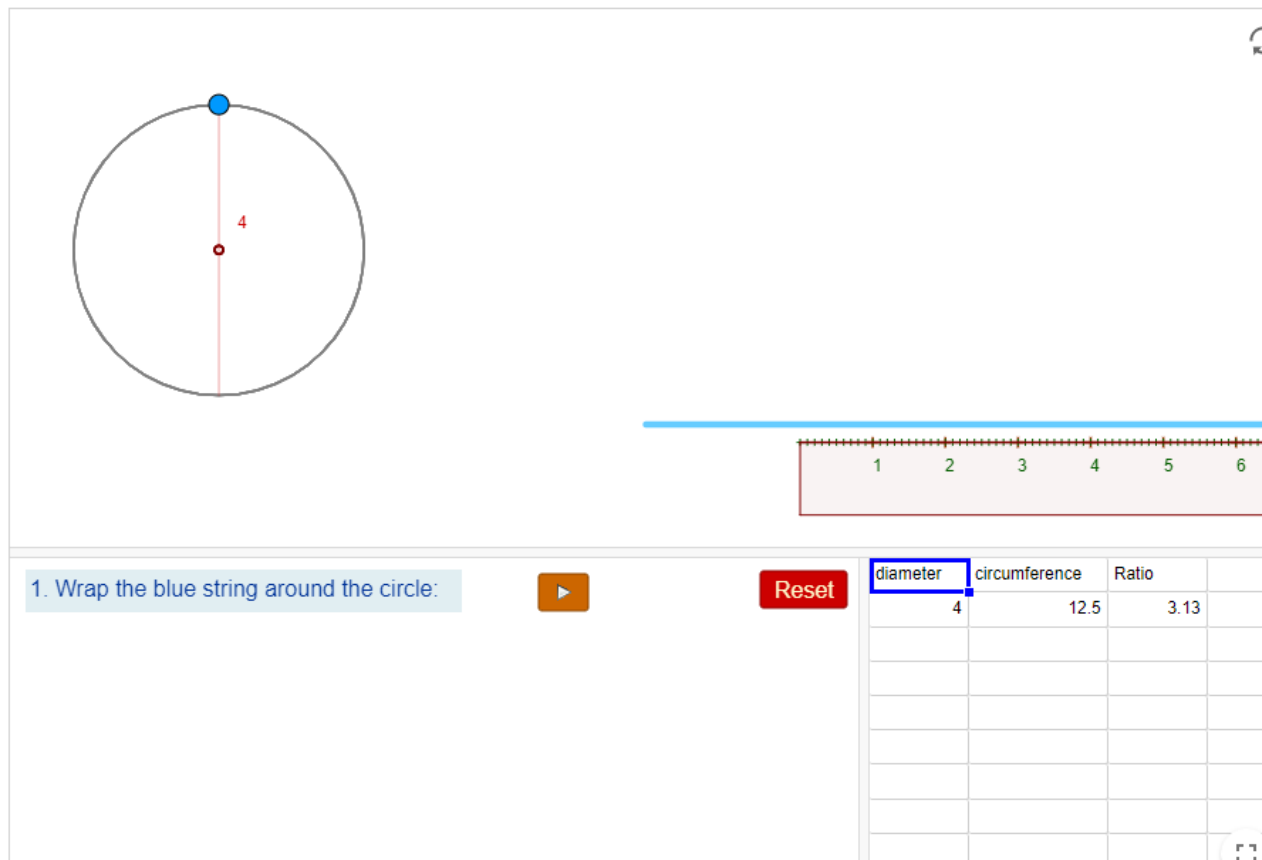
“Many of life's failures are people who did not realize how close they were to success when they gave up.”
— **Thomas A. Edison**


Our greatest glory lies not in never falling, but in rising every time we fall.

P

Pi. The Greek letter pi (π) is used to represent the ratio of the circumference of a circle to its diameter.

Click the image
to explore pi
using
GeoGebra.



1. Wrap the blue string around the circle: 

Reset

diameter	circumference	Ratio
4	12.5	3.13

[Back to Start](#)

P

Place, Value, and Place Value. Place - the name of the place a digit is in within a number; Value - the value a digit holds within a number based on its place; Place-value - the relationship between a digit's place and the value it represents in that place. (*Krystal Shaw*)

Place

Thousands	Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths
	7	3	4	●	8		

In the number 734.8, the digit, 3, is in the tens place.

Value

Thousands	Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths
	7	3	4	●	8		

In the number 734.8, the value of the 3 is three tens or 30.

Place-value

Thousands	Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths
	7	3	4	●	8		
	7	4	3	●	8		

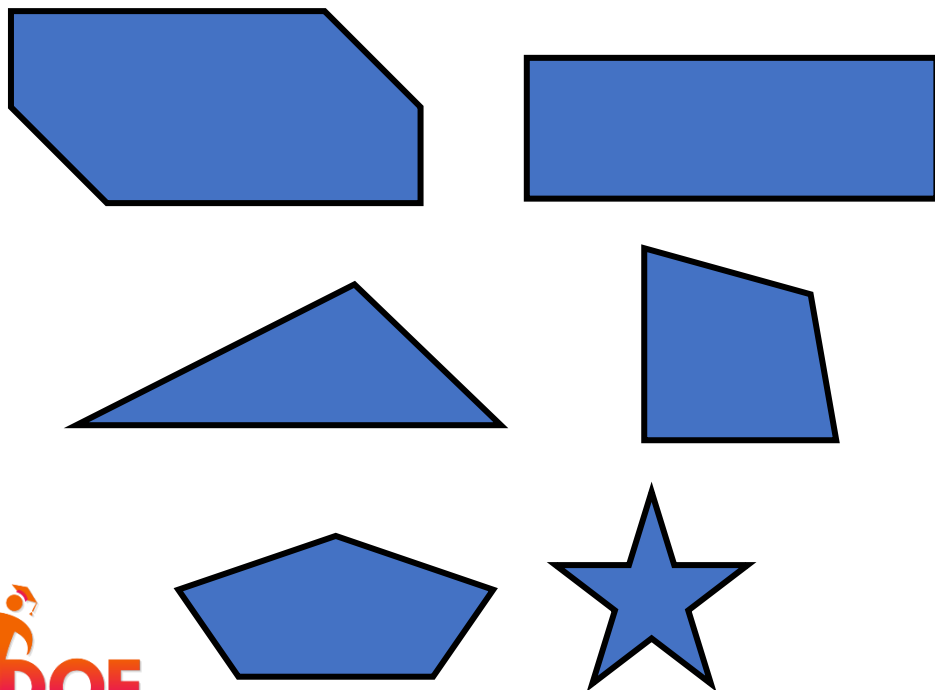
Switching the places of the 4 and the 3 means the places, values, and place-values for these digits has also changed.

P

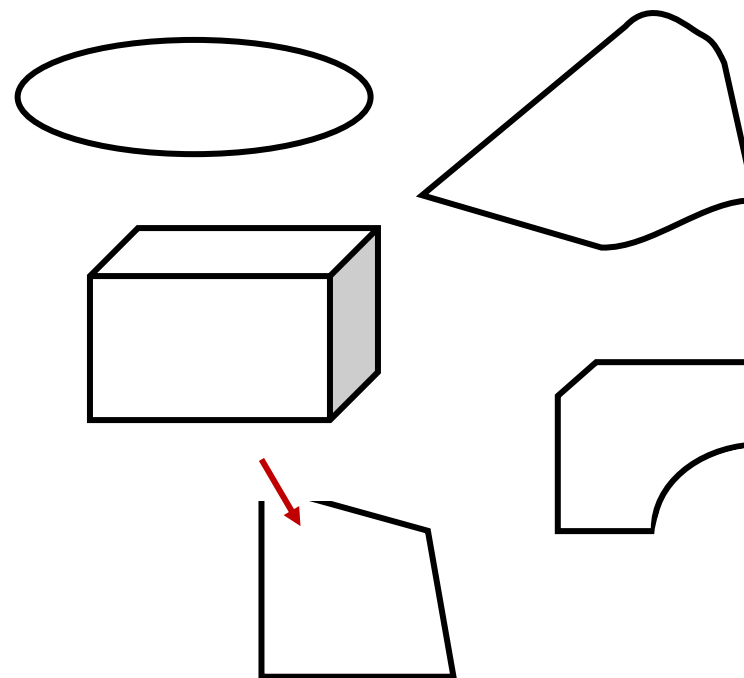
Polygon. A closed plane figure with at least three straight sides and angles. A polygon is regular only when all sides are congruent, and all angles are congruent.

Examples:

Polygons



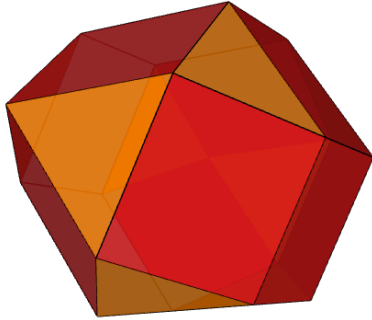
Not Polygons



P

Polyhedron. A three-dimensional shape with flat polygonal faces, straight edges and sharp corners or vertices. (Plural: Polyhedra)

Click the image below to take an interactive view of over 100 polyhedra using the [Polyhedra viewer](#).



The images below show the five regular polyhedra known as the Platonic solids. Regular means that the faces are congruent (identical in shape and size) regular polygons (all angles congruent and all edges congruent), and the same number of faces meet at each vertex. There are only five such Platonic solids.



Image source: [Brilliant.org](#)

P

Polynomial. An expression consisting of variables and coefficients, that involves only the operations of addition, subtraction, multiplication. Variables should have only non-negative integer exponents.

Examples:

Polynomial in one variable:

$$x^2 - 8x + 12$$

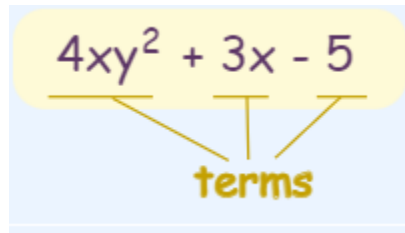
Polynomial in three variables:

$$x^3 - 3xyz^2 - yz + 2$$

P

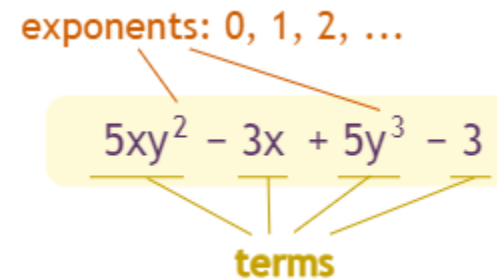
Polynomial Expressions. A polynomial is any expression of the form: $ax^n + bx^{n-1} + cx^{n-2} + \dots + \text{constant}$, where the **exponents**, n , $n - 1$, $n - 2$, etcetera are all **positive integers**. Note that **not all powers need to occur** because the **coefficients**, i.e., a , b , c , etcetera can be zero. See *also*: [Coefficient](#), [Constant](#), [Exponent](#), [Integers](#).

Examples:



The diagram shows the polynomial $4xy^2 + 3x - 5$ inside a yellow rounded rectangle. Three lines extend from the bottom of the rectangle to the text "terms" below it, indicating that the expression consists of three terms: $4xy^2$, $3x$, and -5 .

Example of a polynomial.
This one has 3 terms



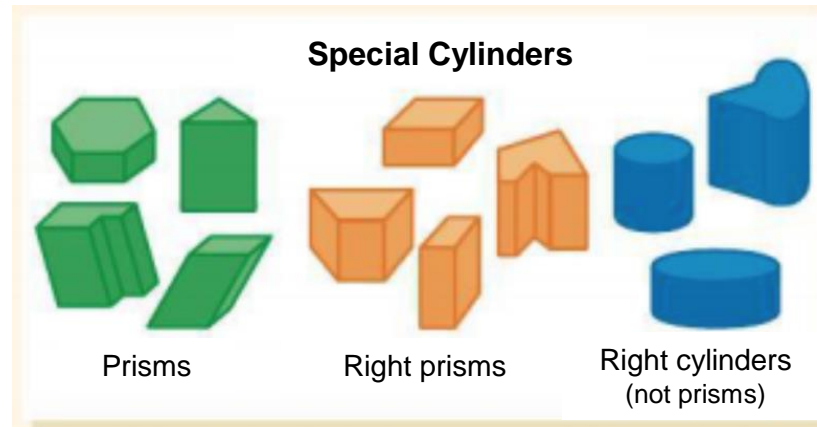
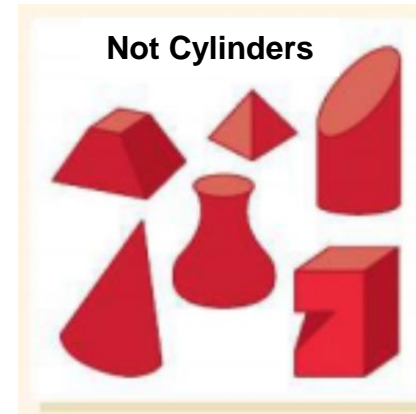
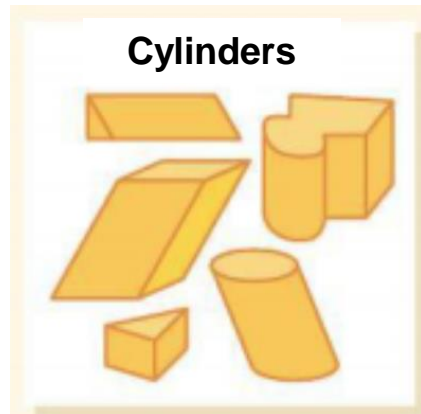
The diagram shows the polynomial $5xy^2 - 3x + 5y^3 - 3$ inside a yellow rounded rectangle. Four lines extend from the bottom of the rectangle to the text "terms" below it, indicating that the expression consists of four terms: $5xy^2$, $-3x$, $5y^3$, and -3 . Above the rectangle, the text "exponents: 0, 1, 2, ..." is written in orange, with lines pointing to the exponents in the terms: 2 for y^2 , 1 for x , 3 for y^3 , and 0 for the constant -3 .

Example of a polynomial.
This one has 4 terms.

P

Prism. A cylinder with polygons for bases. All prisms are special cases of cylinders.

Examples:



P

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Click the image below to engage with this probability number line using

The sun will rise tomorrow morning.

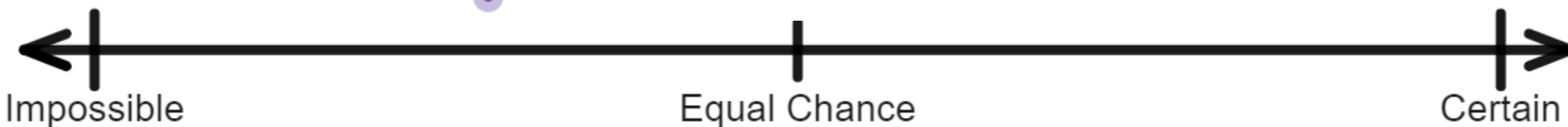
You will win the lottery tomorrow!

A fair, six-sided die is rolled and it will be an even number

It will be hot in Georgia in July.

A tree will blast off to the moon.

A fair coin is flipped and it will land on heads.



P

Probability Distribution. The set of possible values of a random variable with a probability assigned to each. The table or equations showing respective probabilities of different possible outcomes of a defined event or scenario.

Example:

Let's suppose a coin was tossed twice and we have to show the probability distribution of showing heads. The possible outcomes could be (H, H), (H, T), (T, H), (T, T). The possibilities of heads on two flips of a coin would be: 0, 1, or 2 heads.

The probability distribution for selecting "heads" could be shown as:

Number of Possible Outcomes	4		
Number of Heads Tossed	0	1	2
Number of Outcomes	1	2	1
Probability of Event	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

P

Probability Model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. See also: [Uniform Probability Model](#).

Example:

Provide a small plastic cup to each pair of students. Ask them to list the possible ways the cup could land if they tossed it in the air and let it fall on the floor. Which of the possibilities do they think is most (or least) likely? Why? Have students toss the cup 20 times, each time recording how it lands. Students should agree on a uniform method of tossing the cups to ensure unbiased data. Record each pair's data in a class chart. Discuss the differences and generate reasons for them. Have students predict what will happen if they pool their data. Pool the data and compute the three ratios: one for each type of landing (upside down, right side up, or on the side) to the total number of tosses. The relative frequency of the combined data should approximate the actual probability.

P

Procedural Understanding. A surface level of understanding in mathematics computation, where students rely on memorized procedures with little or no understanding of why they work, when to use them, and/or how these procedures might be related to other mathematical ideas. Students who rely solely on procedural understanding in computation often have difficulty seeing relationships between operations and determining the reasonableness of their solutions to problems.

P

Procedure. See: [Computational Algorithm](#), [Computational Strategy](#).



COMPUTATIONAL STRATEGIES FOR WHOLE NUMBERS

Mathematics Place-Value Strategies and US Traditional Algorithms

Specific mathematics strategies for teaching and learning are not mandated by the Georgia Department of Education or assessed on state or federally mandated tests. Students may solve problems in different ways and have the flexibility to choose a mathematical strategy that allows them to make sense of and strategically solve problems using efficient methods that are most comfortable for and makes sense to them. It is critical that teachers and parents remain partners to help each child grow to become a mathematically literate citizen. [These standards preserve and affirm local control and flexibility.](#)

In mathematics, the emphasis is on the reasoning and thinking about the quantities within mathematical contexts. Algorithms, tape diagrams (bar models), and number line representations are a few examples of ways that students communicate their strategic thinking in a written form.

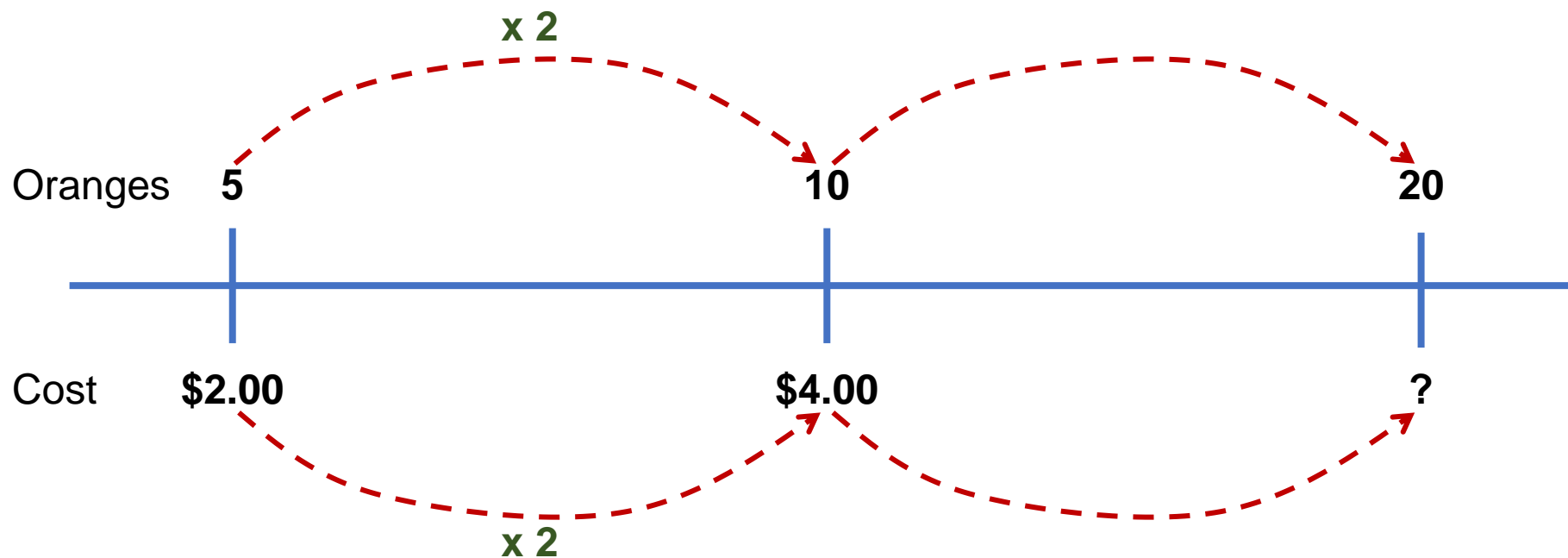


[Back to Start](#)

P

Proportional Relationship. Proportions are the comparison of two equal ratios. Therefore, proportional relationships are relationships between two equal ratios.

Example: Oranges are sold in a bag of 5 for \$2. The ratio of oranges to their cost is 5:2 or $5/2$. We can find the cost of 20 oranges by using proportional reasoning.



P

Pythagorean Theorem. A geometric theorem that states that the sum of the squares on the legs of a right triangle is equal to the square on the hypotenuse.

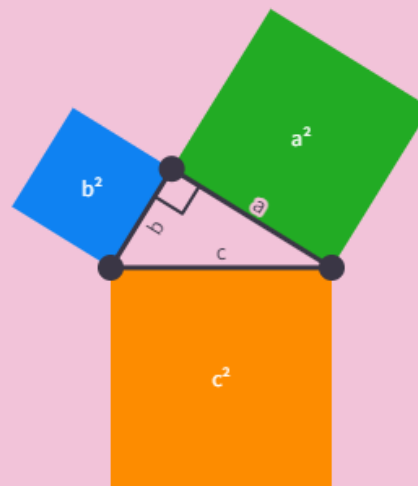
Click the image below to explore the Pythagorean Theorem using Mathigon.

▶ Pythagoras' Theorem

In any right-angled triangle, the square of the length of the **hypotenuse** (the side that lies opposite the right angle) is equal to the sum of the squares of the other two sides. In other words,

$$a^2 + b^2 = c^2$$

The converse is also true: if the three sides in a triangle satisfy $a^2 + b^2 = c^2$, then it must be right-angled.



Q

[Quadratic Function](#)

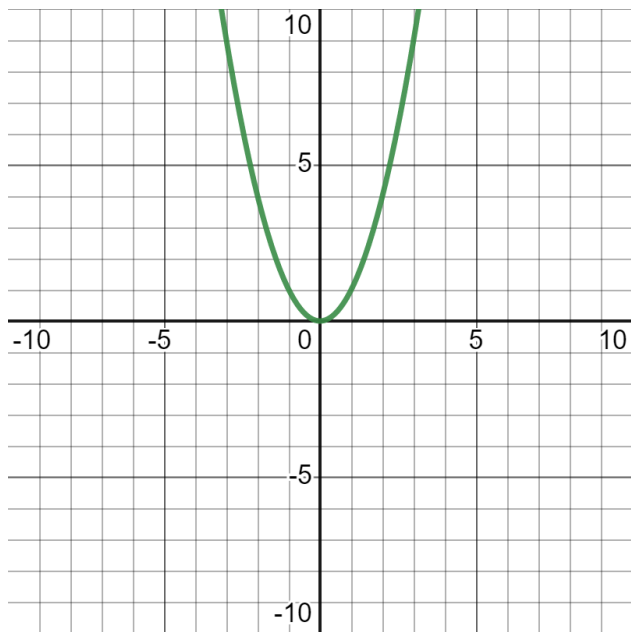
[Quadrilateral](#)

[Quartile](#)

[Quotient](#)

Q

Quadratic Function. A polynomial function with a degree of 2, having at least one term with one or more variables having the highest exponent of 2.



Parent function: $f(x) = x^2$

Quadratic Functions

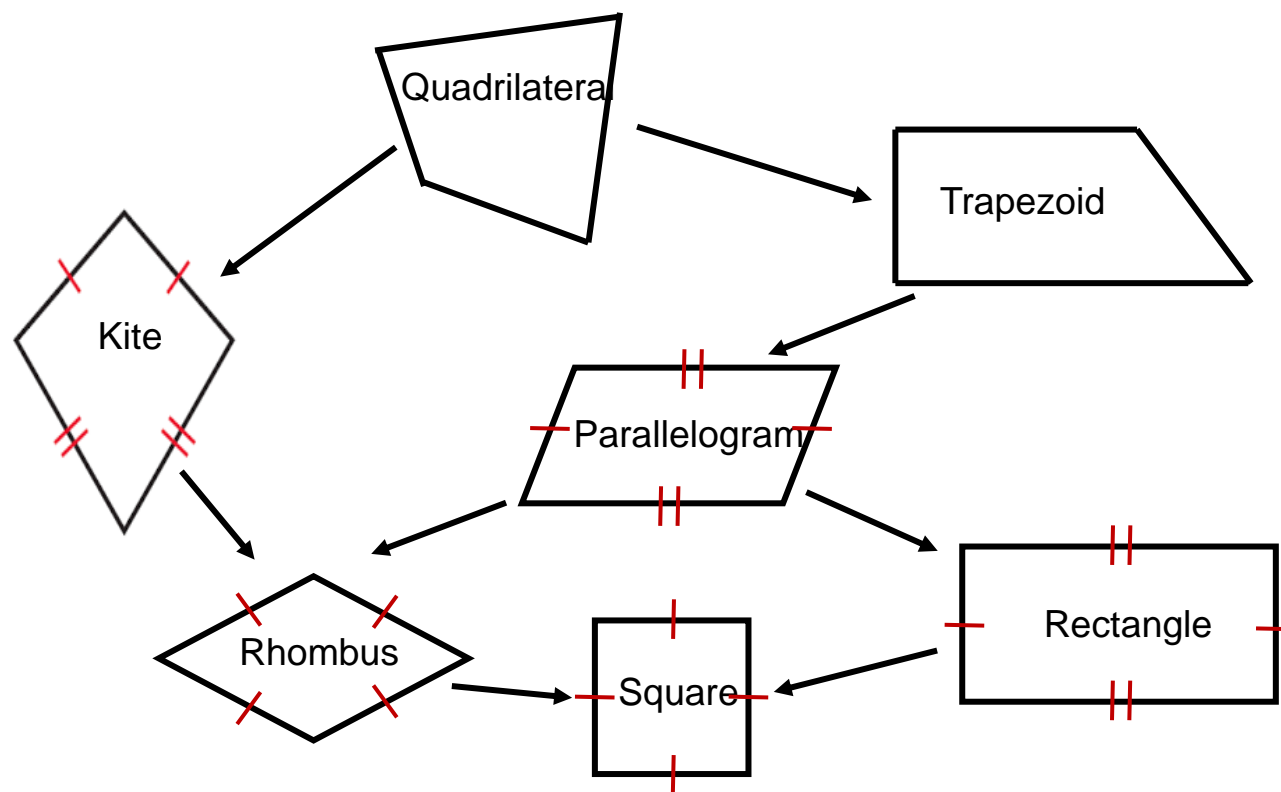
$$f(x) = ax^2 + bx + c, \text{ where } a \neq 0$$

Different Forms of Quadratic Functions

Different Forms of Quadratic Functions		
Standard Form	Vertex Form	Intercept Form (Factored)
$ax^2 + bx + c$	$a(x - h)^2 + k$	$a(x - m)(x - n)$

Q

Quadrilateral. A polygon having four sides and four vertices. See below for examples of quadrilaterals and how they are related.



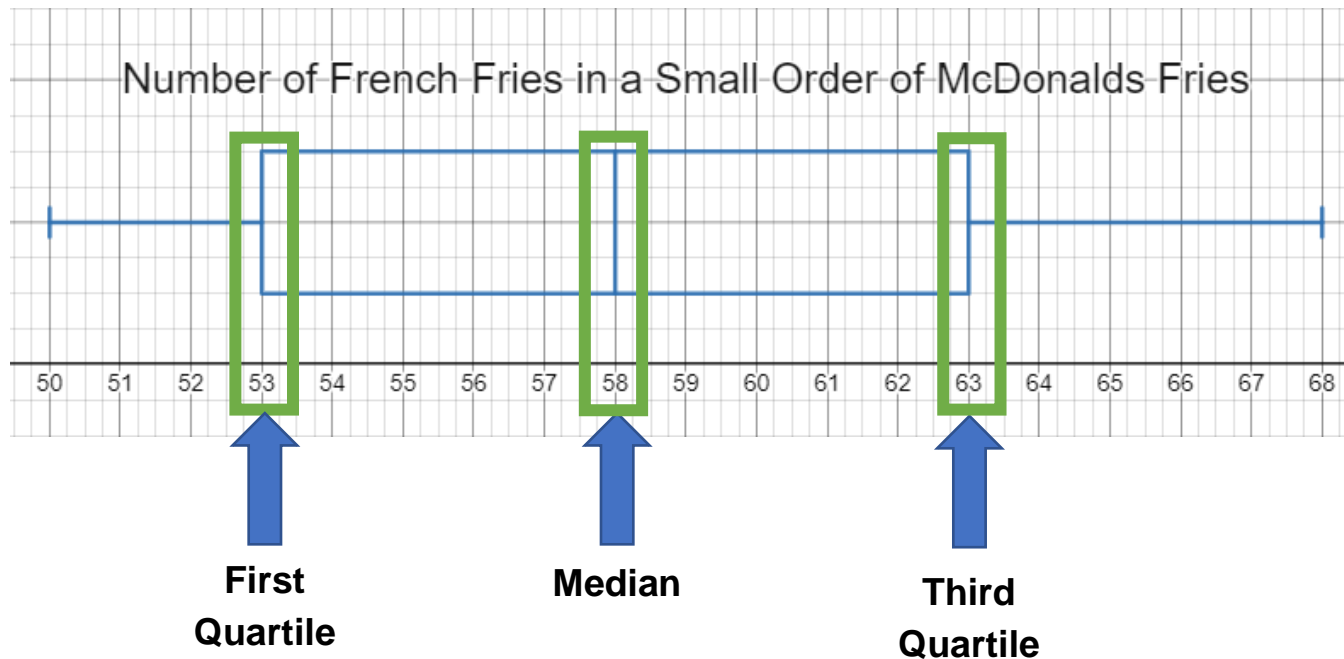
Q

Quartile. An order statistic where the number of data points are divided into four parts, or quarters, of more-or-less equal size. The data must be ordered from smallest to largest to compute quartiles. See also: [Median](#), [First quartile](#), [Third quartile](#), [Interquartile range](#)

Example:

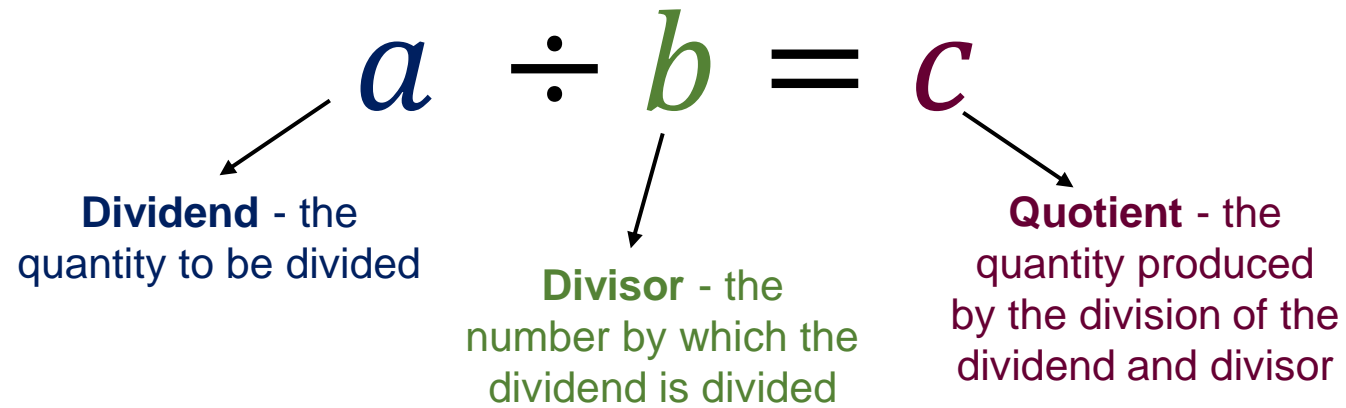
The data below were collected to determine the number of French fries in a small order of fries from McDonalds.

50, 53, 53, 54, 56, 58, 59, 61, 63, 65, 68



Q

Quotient. A quantity produced by the division of two numbers.



Example:

A teacher has 28 students and wants their desks to be in groups of four. How many groups will she make?

$$28 \div 4 = 7$$

R

[Radius](#)

[Random Variable](#)

[Range \(Algebra\)](#)

[Range \(Statistics\)](#)

[Ratio](#)

[Rational Expression](#)

[Rational Number](#)

[Reason](#)

[Reason Quantitatively](#)

[Rectangle](#)

[Rectangular Array](#)

[Relative Frequency](#)

[Relational Thinking](#)

[Repeating Decimal](#)

[Rhombus](#)

[Right Angle](#)

[Rigid Motion](#)

[Rigor](#)

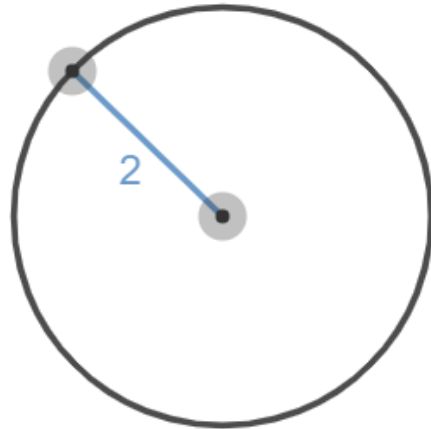


R

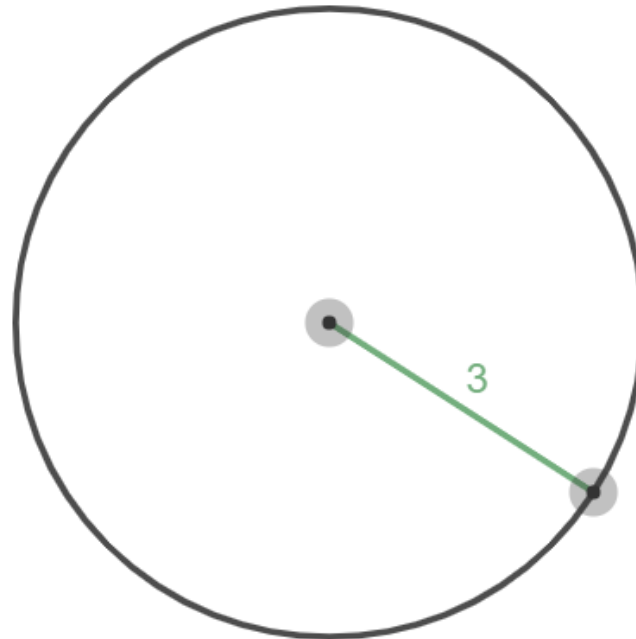
Radius. The distance from the center of a circle to any point on its circumference. See *also*: [Circle](#), [Circumference](#), [Diameter](#), [Chord](#). *Plural: Radii*

Examples:

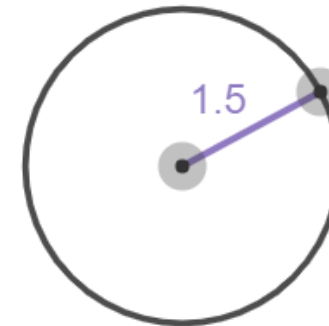
This circle has a radius of 2 units.



This circle has a radius of 3 units.

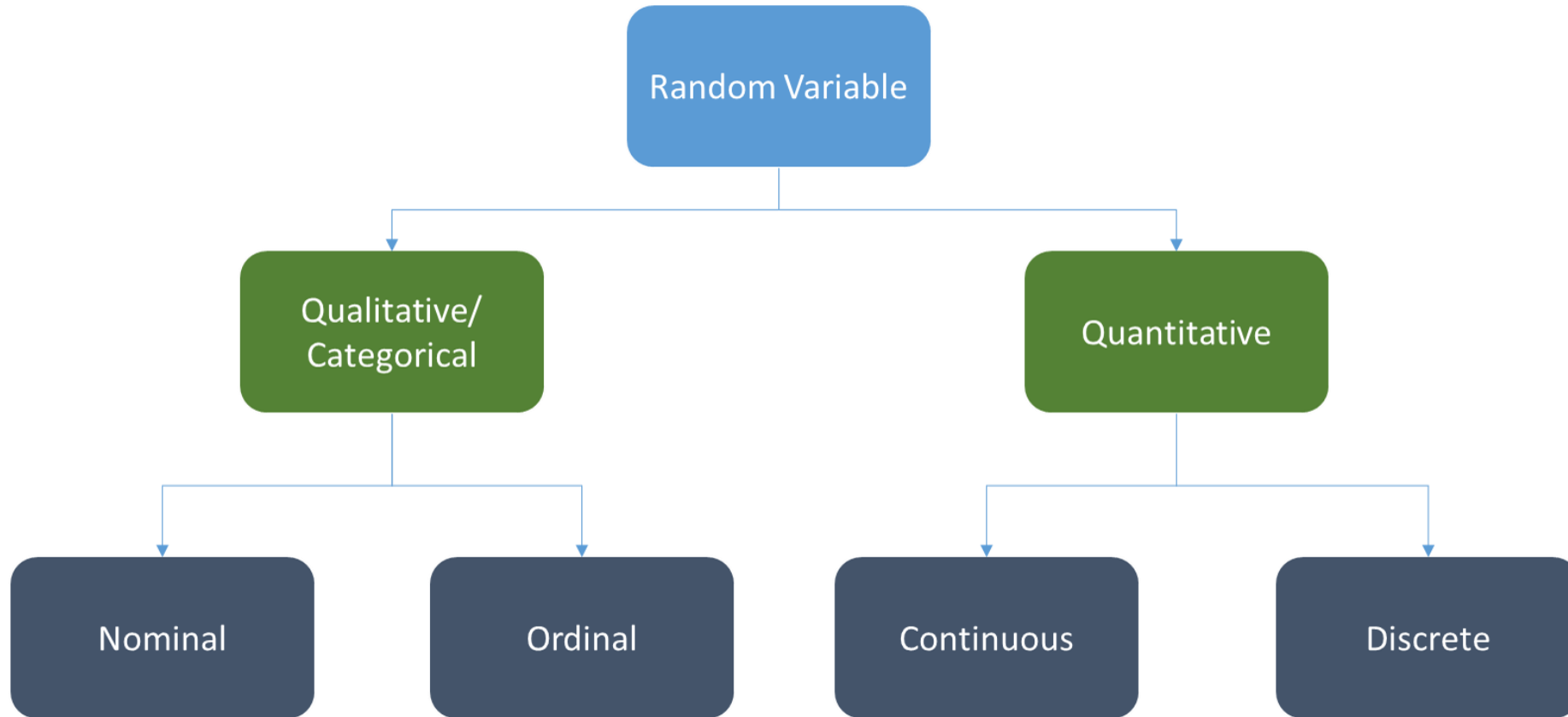


This circle has a radius of 1.5 units.



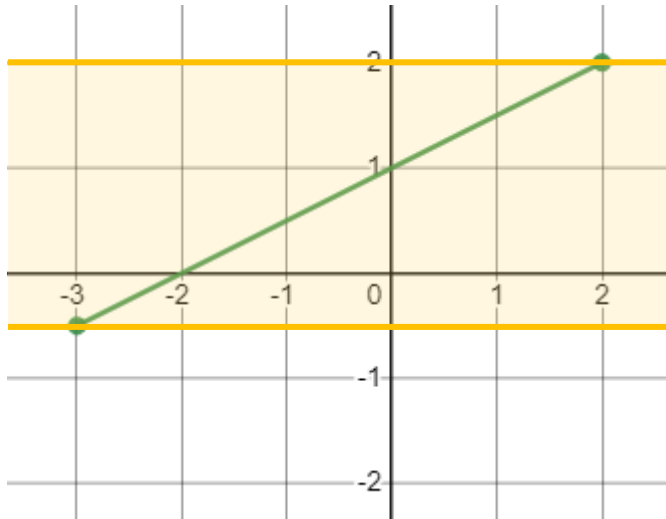
R

Random Variable. An assignment of a numerical value to each outcome in a sample space.

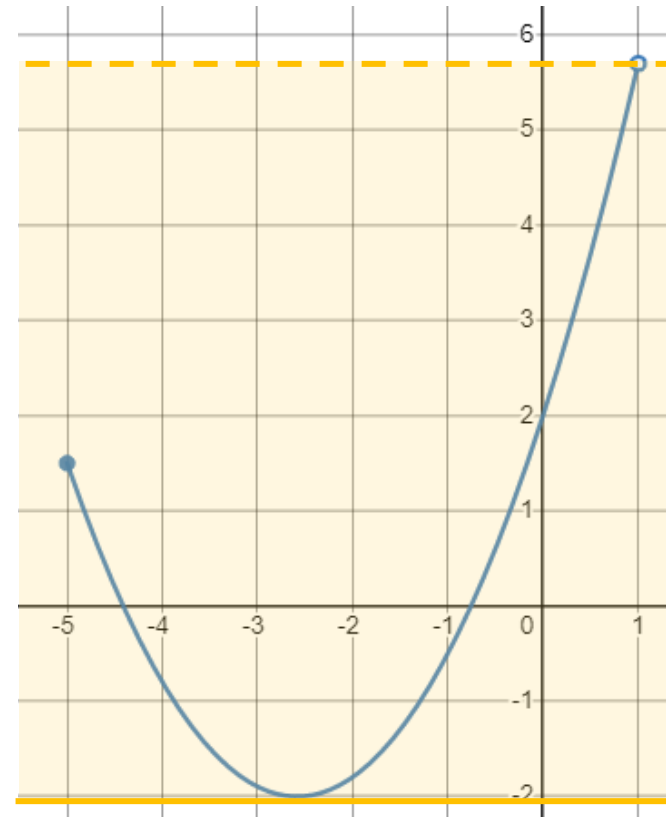


R

Range (Algebra). The set of outputs the function achieves when it is applied to its whole set of outputs. See also: [Domain](#)



The range for this function is
 $\{-0.5 \leq y \leq 2\}$



The range for this function is
 $\{-2 \leq y < 5.7\}$

R

Range (Statistics). The difference between the lowest and highest values in a set of data.

Example:

The data below were collected to determine the number of French fries in a small order of fries from McDonalds.

50, 53, 53, 54, 56, 58, 59, 61, 63, 65, 68

**To determine the range for this data set,
subtract the minimum value from the maximum value.**

$$68 - 50 = 18$$

The range for this data set is 18 French fries.

R

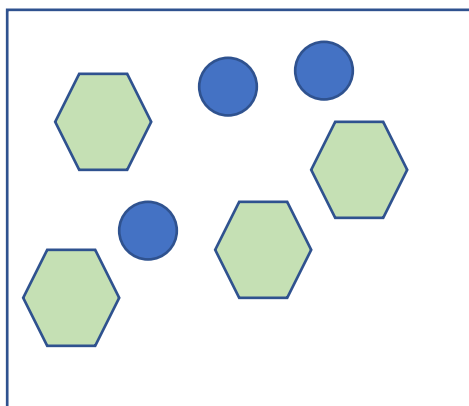
Ratio. A number that relates two quantities or measures within a given situation in a multiplicative relationship. There are four types of ratios: Part-to-Part, Part-to-Whole, Ratios as Quotients, and Ratios as Rates.

Part-to-Part Ratios

The ratio of hexagons to circles is 4 to 3 (4:3). This is not a fraction even though it may be written using fraction notation.

Part-to-Whole Ratios

The ratio of hexagons shapes is 4 to 7 (4:7). This can be written as $\frac{4}{7}$ and can be thought of as four sevenths of the shapes (a fraction).



Ratios as Quotients

If apples are 4 for \$1.00, the ratio of money to apples is \$1.00 to 4 apples. Dividing gives a quotient of \$0.25 per apple. This is the unit rate.

Ratios as Rates

Rates involve two different units and how they relate to each other (*Miles per gallon, passengers per busload, roses per bouquet, etc.*). Relationships between two units of measure are also rates (*inches per foot, centimeters per inch, feet per second, etc.*).

R

Rational Expression A quotient of two polynomials with a non-zero denominator. Or in other words, it is a fraction whose numerator and denominator are polynomials.

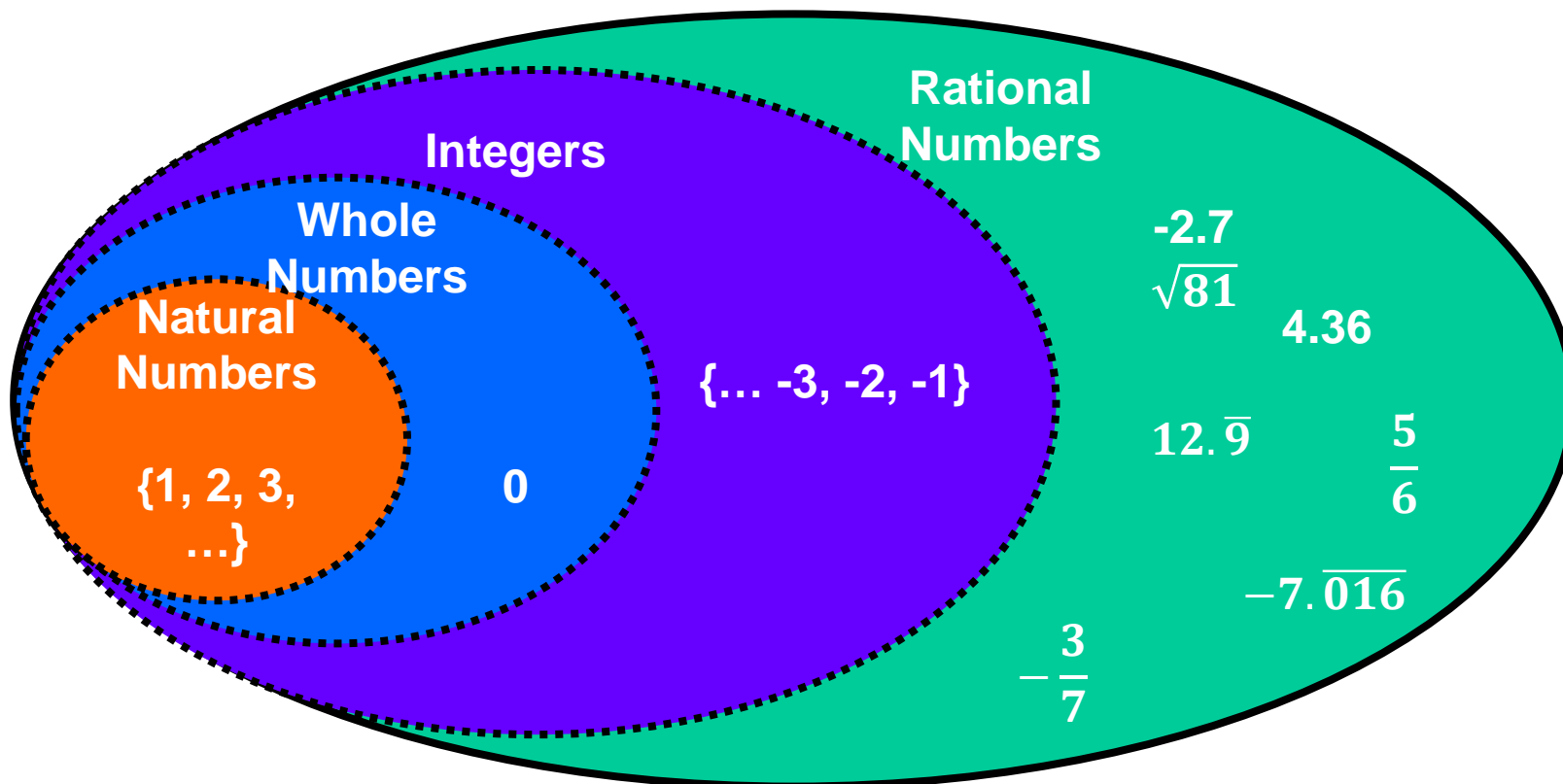
Examples:

- $\frac{1}{x}$
- $\frac{x+5}{x^2-4x+4}$
- $\frac{x(x+1)(2x-3)}{x-6}$

Notice that the numerator can be a constant and that the polynomials can be of varying degrees and in multiple forms.

R

Rational Number. A number expressible in the form $\frac{a}{b}$ or $-\frac{a}{b}$ for some fraction $\frac{a}{b}$. The rational numbers include the integers. The image below shows some examples of rational numbers.



R

Reason(n). The why behind a mathematical idea.

Reason (v). To make sense of the why underlying a mathematical idea.

K	1	2	3	4	5	6	7	8	HS Algebra: Concepts & Connections	HS Geometry: Concepts & Connections	HS Advanced Algebra: Concepts & Connections
Mathematical Modeling (MM)											
Mathematical Practices (MP)											
Data & Statistical Reasoning (DSR)											
Numerical Reasoning (NR)											
Patterning & Algebraic Reasoning (PAR)											
Geometric and Spatial Reasoning (GSR)											
Measurement & Data Reasoning (MDR)											
									Functional & Graphical Reasoning (FGR)		
							Probability Reasoning (PR)		Probabilistic Reasoning (PR)		

R

Reason Quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Connections to Classroom Practices (13)

KINDERGARTEN (2 CLIPS)	+
3RD GRADE (1 CLIP)	+
4TH GRADE (1 CLIP)	+
5TH GRADE (3 CLIPS)	+
5TH/6TH GRADE (1 CLIP)	+
6TH GRADE (2 CLIPS)	+
8TH GRADE (2 CLIPS)	+
9TH-12TH GRADE (1 CLIP)	+



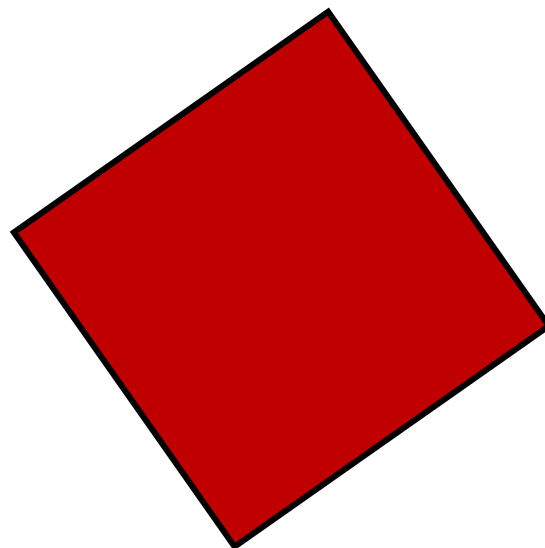
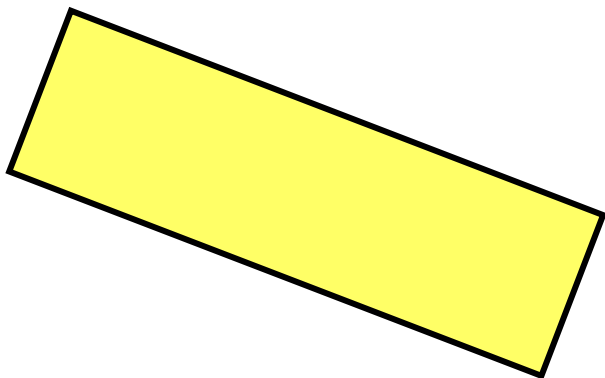
Source: Charles A. Dana Center

[Back to Start](#)

R

Rectangle. A parallelogram with four right angles. See *also*: [Polygon](#), [Quadrilateral](#).

Examples:



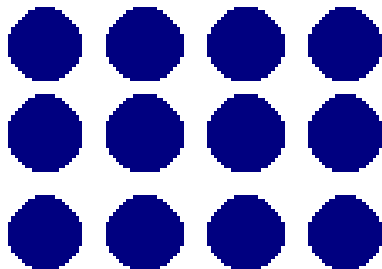
R

Rectangular Array. A set of objects arranged into rows and columns. Each row must contain the same number of objects as other rows, and each column must contain the same number of objects as other columns.

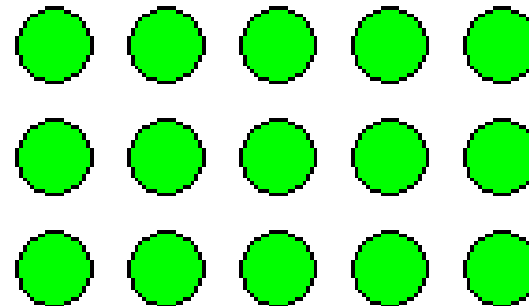
Examples:

Click the images below to learn more about using arrays.

Using Arrays to Explore Numbers



Illustrating Number Properties with Arrays



R

Relative Frequency. The number of times an event occurs divided by the total number of possible outcomes.

Example:

A group of 90 students participates in sports. 36 play football, 68 are on the track team. What is the relative frequency of a student playing both sports?

Possible solution: There are $68 + 36 = 104$ total teammates. 90 students play at least one sport.

$104 - 90 = 14$ students play two sports. The relative frequency of a student playing both sports is

$$\frac{14}{90} = \frac{7}{45}$$

R

Relational Thinking. Using the fundamental properties of number and operations to transform mathematical expressions, rather than simply calculating an answer by following a prescribed sequence of procedures. Relational thinking is the foundation of algebra and algebraic thinking.

Students who use relational thinking are able to compare mathematical expressions without calculating:

True or False?

$$5 \times 84 = 10 \times 42$$

True! 10 is double 5 and 42 is half of 84.

$$674 - 389 = 664 - 379$$



True! There is a common difference. 664 is 10 less than 674 and 379 is 10 less than 389.

$$64 \div 14 = 32 \div 28$$

False! 32 is half of 64 and 28 is double 14. The dividend and divisor should both be halved or doubled for this to be true.

R

Repeating Decimal. A number whose decimal representation eventually becomes periodic (i.e., the same sequence of digits repeats indefinitely). The repeating portion of a decimal expansion is conventionally denoted with a vinculum ($\bar{\quad}$). For example, $\frac{1}{3} = 0.3333 \dots = 0.\bar{3}$. See also: [Decimal](#), [Terminating Decimal](#).

Examples of Repeating Decimals:

$$\frac{1}{3} = 0.3333 \dots = 0.\bar{3}$$

$$\frac{1}{11} = 0.090909 \dots = 0.\overline{09}$$

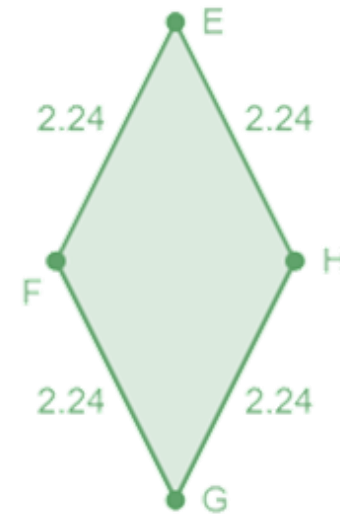
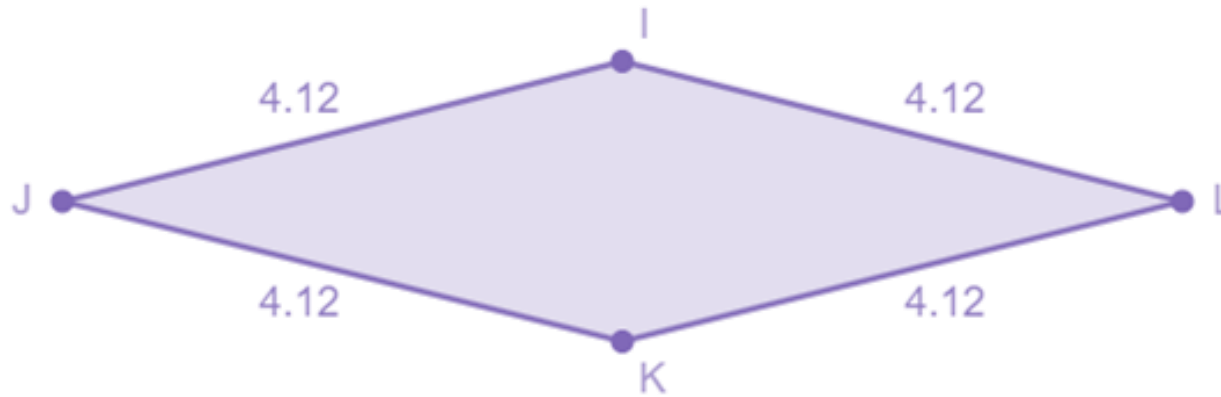
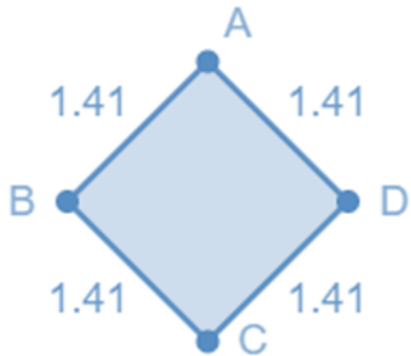
$$\frac{5}{6} = 0.83333 \dots = 0.8\bar{3}$$

$$\frac{2}{7} = 0.285714285714 \dots = 0.\overline{285714}$$

R

Rhombus. A parallelogram with four sides of equal length. See *also*: [Polygon](#), [Quadrilateral](#).

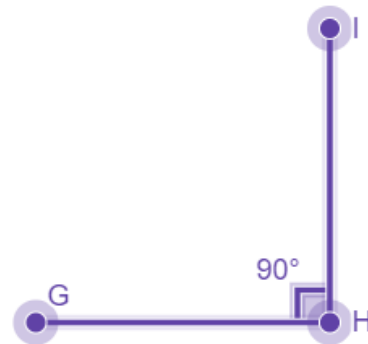
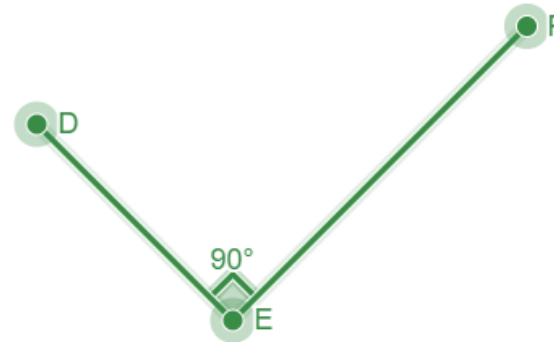
Examples:



R

Right Angle. An angle measuring exactly 90° . See *also*: [Angle](#), [Obtuse Angle](#), [Acute Angle](#), [Perpendicular](#)

Examples: $\angle ABC$, $\angle DEF$, and $\angle GHI$ are all right angles.

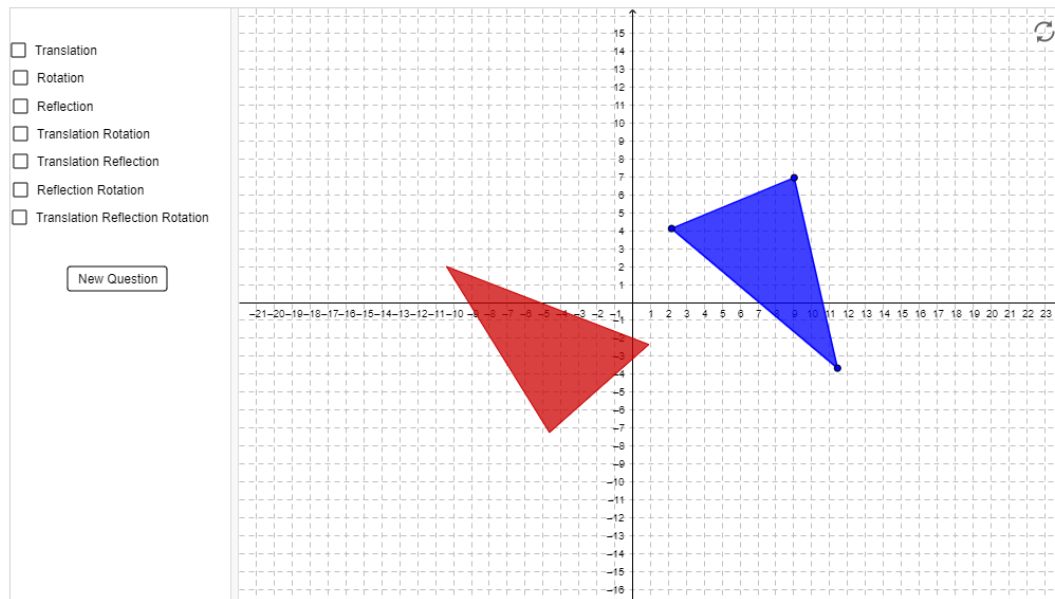


R

Rigid Motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Examples:

Rigid Motion



Click the image above to explore rigid motion using GeoGebra.

Transformation Golf: Rigid Motion



Click the image above to explore rigid motion using Desmos.

R

Rigor. Mathematical rigor is the depth of interconnecting concepts and the breadth of supporting skills students are expected to know and understand.

Rigor in mathematics teaching and learning means focusing with equal intensity on students' conceptual understanding, procedural fluency, and ability to apply what they know to real-world, problem-solving situations.

Mathematical rigor is the effective, ongoing interaction between teacher instruction and student reasoning and thinking about concepts, skills, and challenging tasks that results in a conscious, connected, and transferable body of valuable knowledge for every student.

S

[Sample Space](#)

[Sampling Variability](#)

[Scale](#)

[Scatter Plot](#)

[Separate Problems](#)

[Similarity Transformation](#)

[Slope](#)

[Sphere](#)

[Square](#)

[Square Number](#)

[Square Root](#)

[Standard Algorithm](#)

[Standard Deviation](#)

[Subitize](#)

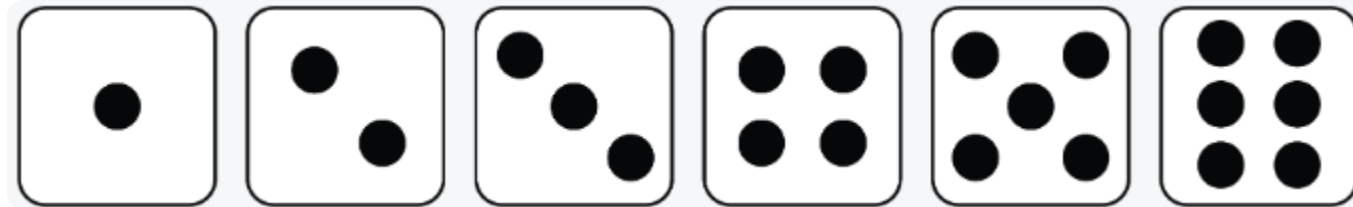
[Supplementary Angles](#)

[Symmetry](#)

S

Sample Space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Example: The sample space of rolling a fair six-sided die is shown below.



S

Sampling Variability (also referred to as **Sample-to-Sample Variability**). The notion that the variability of a sample will differ from another sample of the same population.

Example: In one random sample of 30 turtles the sample mean may turn out to be 350 pounds. In another random sample, the sample mean may be 345 pounds. In yet another sample, the sample mean may be 355 pounds.

Sample 1 Mean: 350 pounds



Sample 2 Mean: 345 pounds



Sample 3 Mean: 355 pounds



Source: [statology.org](https://www.statology.org)

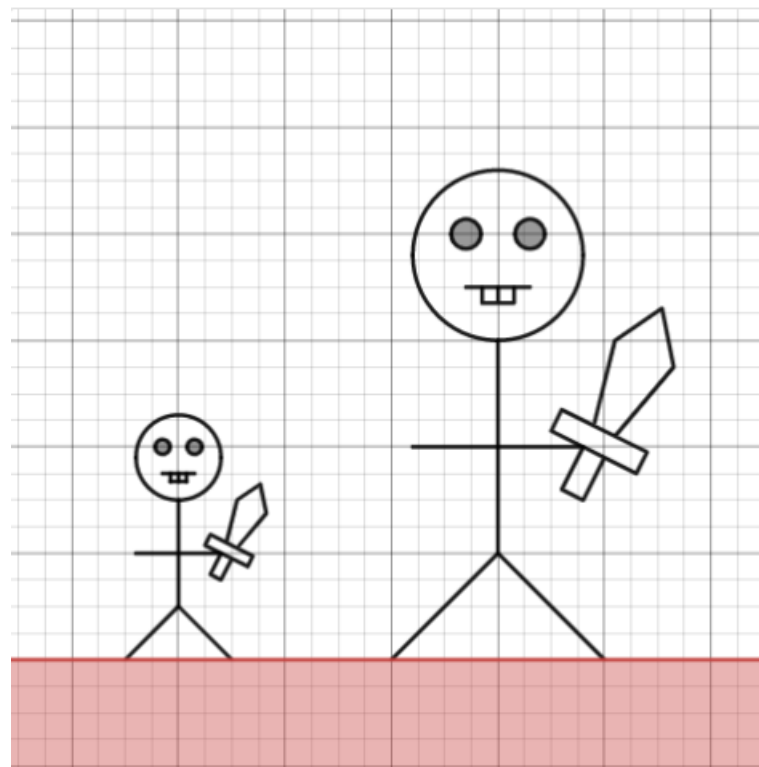
S

Scale. The size of one object in relation to other objects in a design, drawing, or model.

Example:

An image is a scale drawing of the original if the shape is stretched in a way that does not distort it.

The scale factor in this example is 2 because every part of the larger stick figure is twice as large as the corresponding part in the smaller figure.

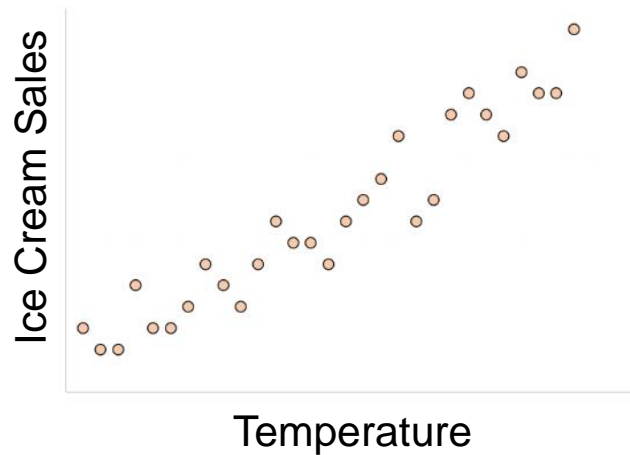


S

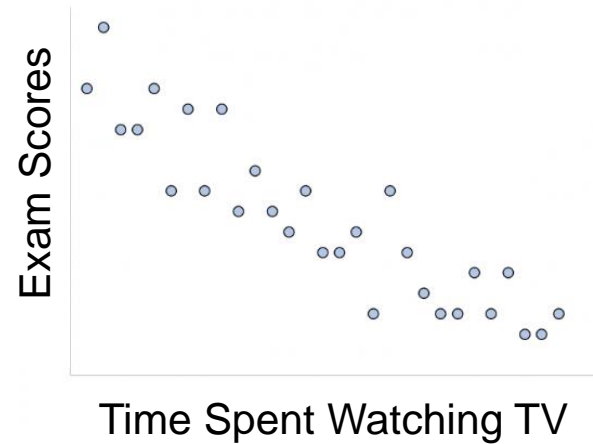
Scatter Plot. A graph in the coordinate plane representing a set of bivariate data.

Examples:

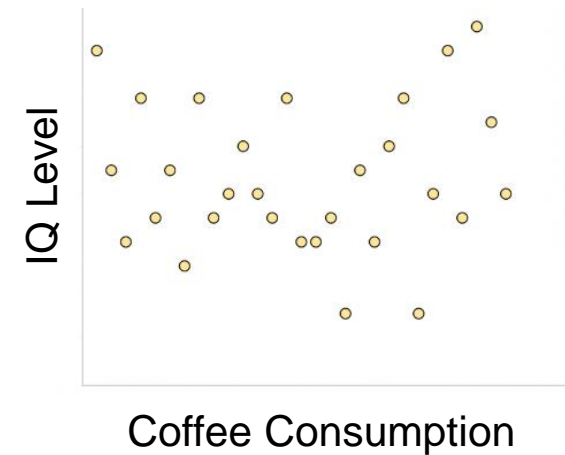
Positive Correlation



Negative Correlation



No Correlation



S

Separate Problems. Separate problems involve a direct or implied action in which a set is decreased by a particular amount. Separate problems can be categorized into three structures: result unknown, change unknown, and start unknown. See also, [Problem Types](#)

Examples:

Result Unknown

*Whitney had 12 puppies.
She gave 3 puppies to
her neighbors. How
many puppies does
Whitney have left?*

Change Unknown

*Whitney had 12 puppies.
She gave some puppies
to her neighbors. Then
she had 9 puppies left.
How many puppies did
Whitney give to her
neighbors?*

Start Unknown

*Whitney had some
puppies. She gave three
puppies to her neighbors.
Then she had 9 puppies
left. How many puppies
did Whitney have to start
with?*

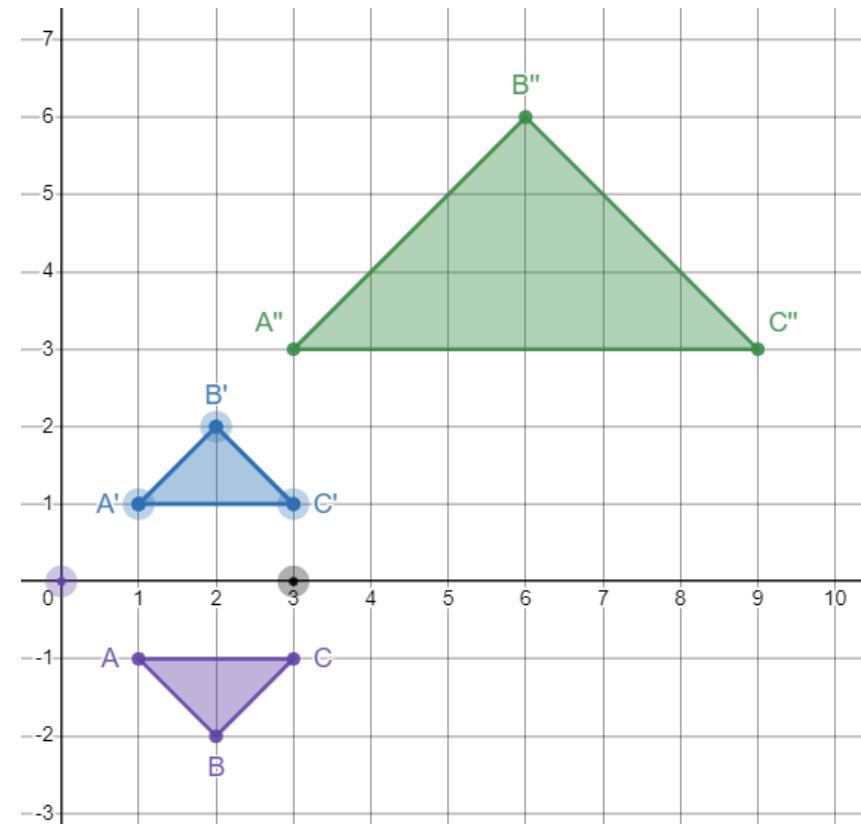
S

Similarity Transformation. A rigid motion followed by a dilation. See also: [Rigid Motion](#), [Dilation](#)

Example:

In the image to the right, **triangle ABC** is reflected across the x-axis, giving **triangle A'B'C'**. These two triangles are congruent because a reflection is a rigid motion.

Next, **triangle A'B'C'** is dilated using a scale factor of 3, giving **triangle A''B''C''**. **Triangle A''B''C''** is similar to both of the smaller triangles.

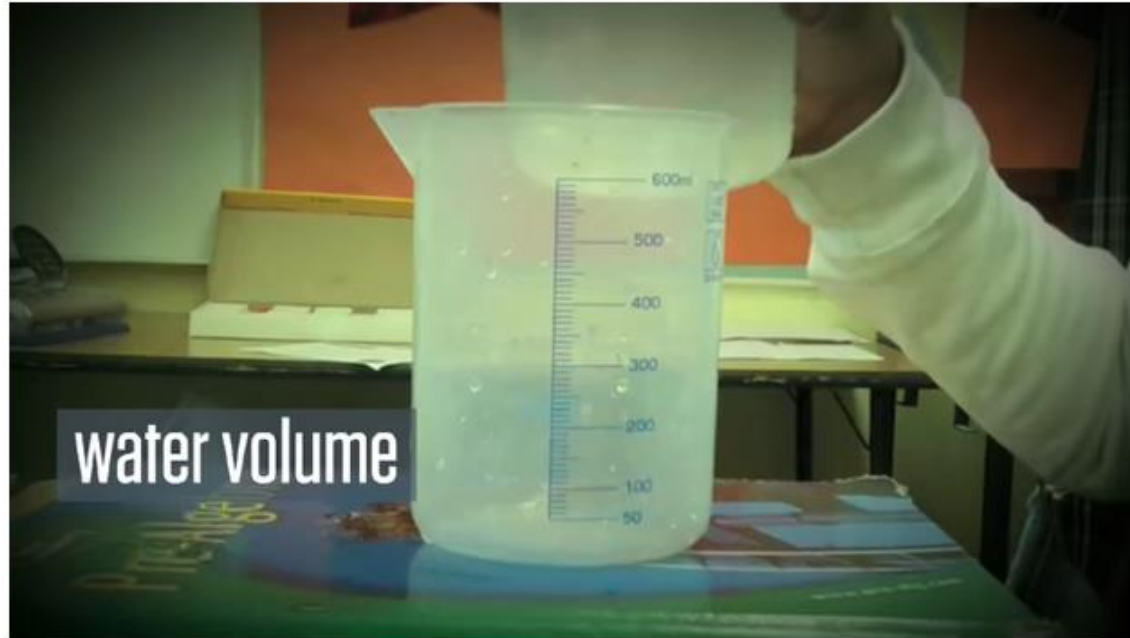


[Back to Start](#)

S

Slope. The rate of change of a function. The slope of a line can be determined by calculating the rise divided by the run of a graph ($m = \frac{\text{rise}}{\text{run}}$).

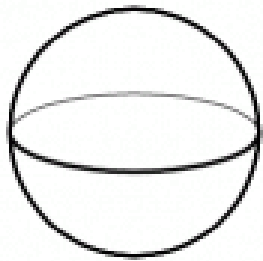
Click the image on the right to see a real-life graph showing the rate of change of the water level as slope.



S

Sphere. A round, solid figure. Every point on the sphere is equidistant from the center of the sphere.

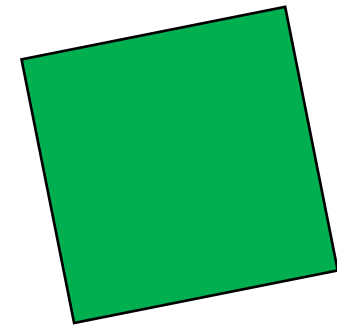
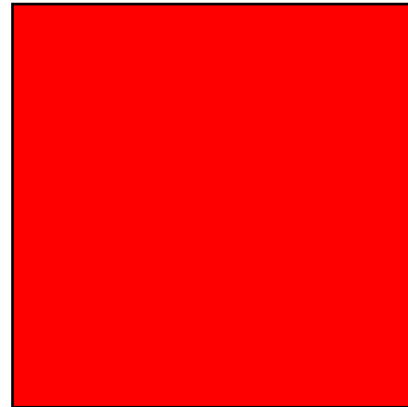
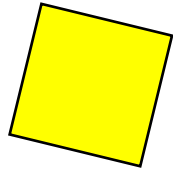
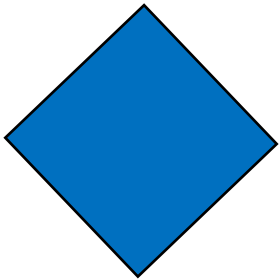
Examples:



S

Square. A plane figure with four equal straight sides and four right angles. See *also*: [Polygon](#), [Quadrilateral](#).

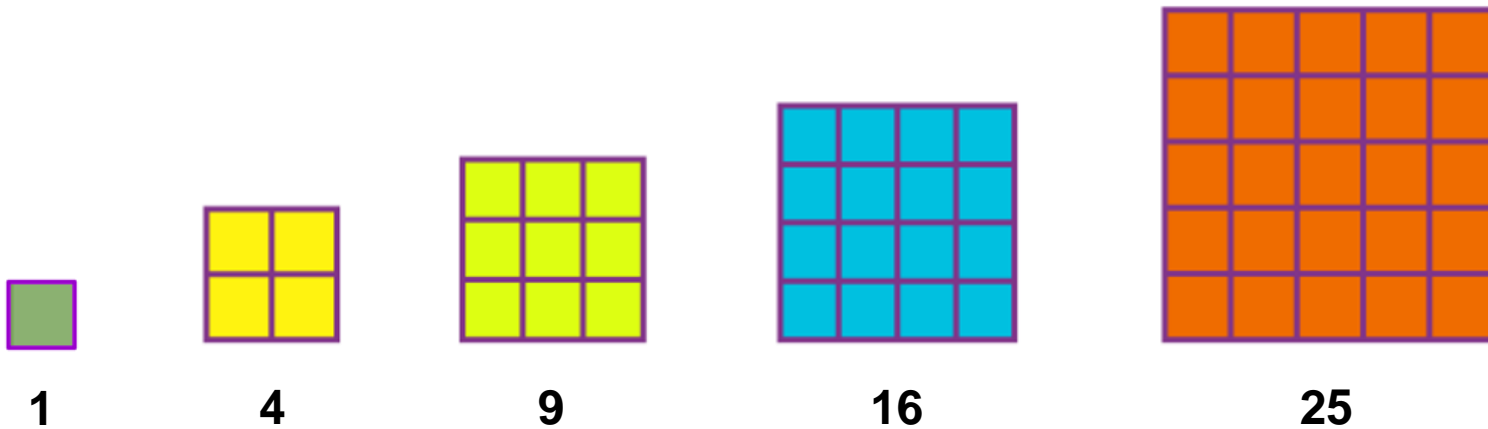
Examples:



S

Square Number. A square number, also known as a perfect square, is created by multiplying an integer by itself. The product or area of the square is the square number.

Examples: Square numbers can be visualized by constructing squares using square tiles. The square numbers below are determined by the number of square tiles in each square.

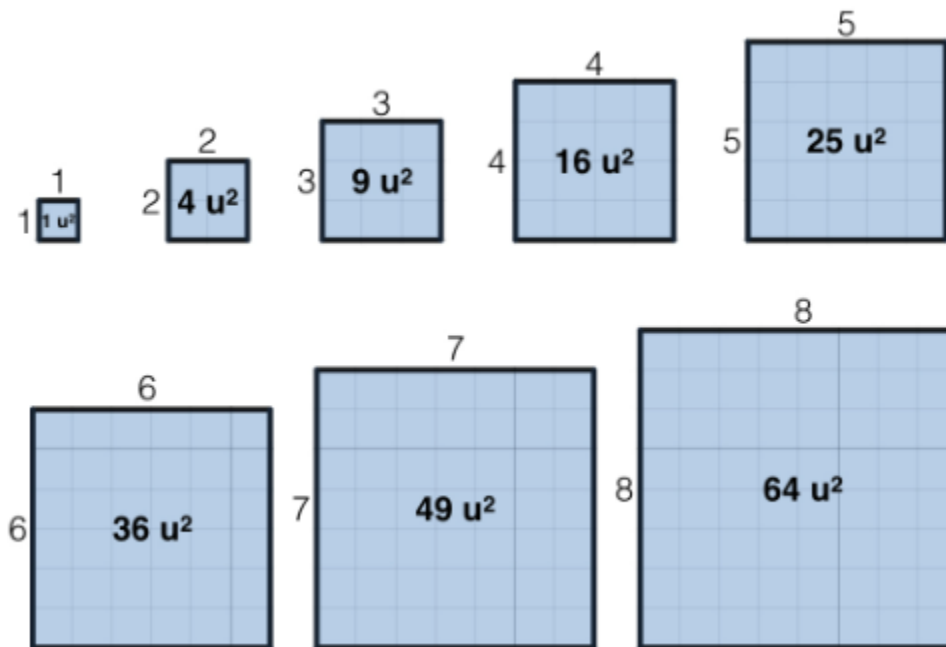


S

Square Root. The square root of a number x , is a number, y such that $y^2 = x$; in other words, the square root of a number is a number whose square is $y \cdot y$.

Examples: Square roots of numbers can be visualized by constructing squares using square tiles. The square numbers below are determined by the number of square tiles in each square. The square roots are represented by the length of a side of the square.

Click the image to the right to explore square roots with your students using Desmos.

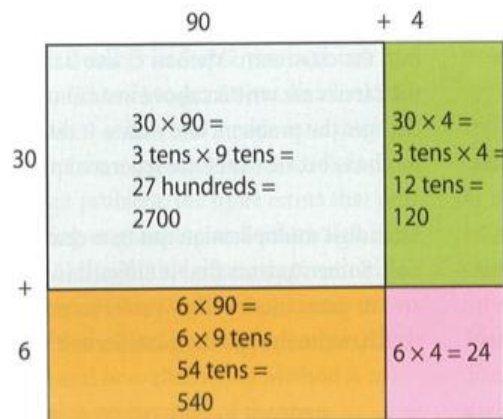


S

Standard Algorithm. An algorithm based on place-value decomposition. (Note: The standard algorithm is not exclusive to the US Traditional algorithm.) See also: [Composition/Decomposition of Number](#).

Examples:

Array/area drawing for 36×94



$$36 \times 94 = (30 + 6)(90 + 4)$$

$$= 30 \times 90 + 30 \times 4 + 6 \times 90 + 6 \times 4$$

Showing the partial products

$$\begin{array}{r} 94 \\ \times 36 \\ \hline 24 \\ 540 \\ 120 \\ 2700 \\ \hline 3384 \end{array}$$

thinking:

6×4

$6 \times 9 \text{ tens}$

$3 \text{ tens} \times 4$

$3 \text{ tens} \times 9 \text{ tens}$

Recording the carries below for correct place value placement

$$\begin{array}{r} 94 \\ \times 36 \\ \hline \overset{5}{2} \overset{2}{4}4 \\ \overset{2}{1} \\ \hline 720 \\ \hline 3384 \end{array}$$

0 because we are multiplying by 3 tens in this row

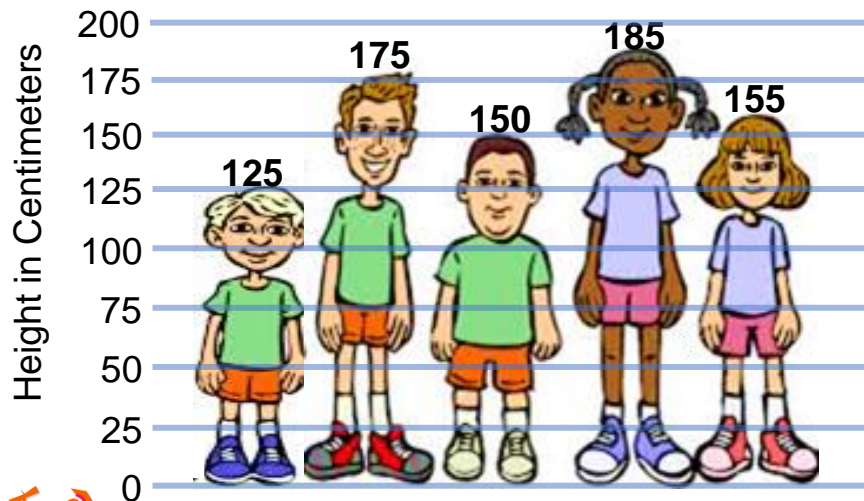
	90	+ 4	
30	2700	120	2700
+			540
6	540	24	120
			<u>24</u>
			3384

S

Standard Deviation. A measure of the amount of variation of a set of data values. A low standard deviation indicates that the values tend to be close to the mean of the data. A high standard deviation indicates that the values are spread out over a wider range. See *also*: [Mean](#), [Range](#), [Variance](#).

Example:

Five friends measured their height to the nearest cm as shown below.



① Find the mean of the friends' heights.

$$\begin{aligned} \text{Mean} &= \frac{125+150+155+175+185}{5} \\ &= 158 \text{ cm} \end{aligned}$$

② Find the distance from the mean for each person's height:

$$\begin{aligned} 158-125 &= 33 \text{ cm} \\ 158-150 &= 8 \text{ cm} \\ 158-155 &= 3 \text{ cm} \\ 175-158 &= 17 \text{ cm} \\ 185-158 &= 27 \text{ cm} \end{aligned}$$

③ Find the variance: Take each difference and square it, then find the average:

$$\frac{33^2+8^2+3^2+17^2+27^2}{5} = 436$$

④ The standard deviation is the square root of the variance:

$$\begin{aligned} \sqrt{436} &= 20.88\dots \\ &= 21 \text{ (to the nearest cm)} \end{aligned}$$

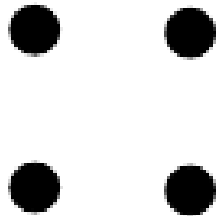
⑤ Which heights are within one standard deviation (21 cm) of the mean?

S

Subitize. Quickly recognize and name the number of objects in a group without counting.

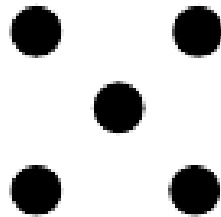
Examples:

How many do you see? How do you see them?



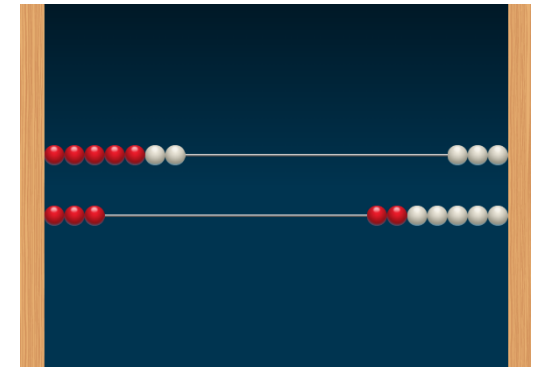
I see four in a square!

I see two on top and two on the bottom and that makes four!



I see four in a square and one in the middle and that makes 5!

I see three slanted and two more. 3 and 2 more makes 5!



I see 7 and 3 more makes 10!

I see 6 and 4 more makes 10!

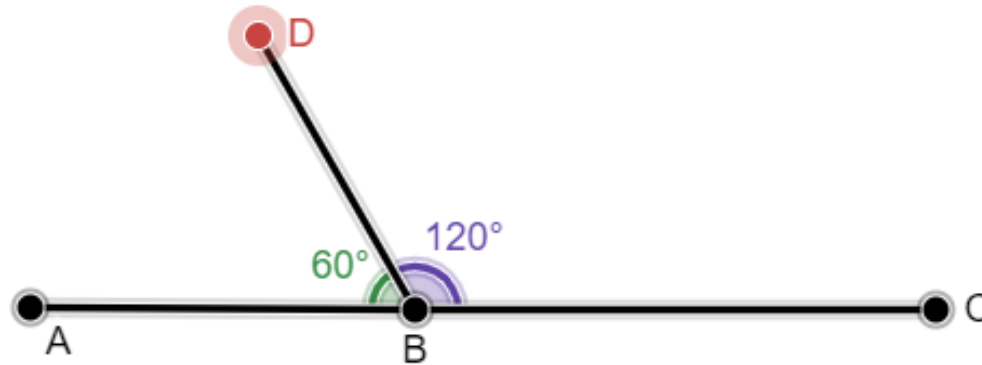
S

Supplementary Angles. Two angles are supplementary when their sum is 180° .

Example:

In the image below, $\angle ABD$ and $\angle DBC$ are supplementary because $60^\circ + 120^\circ = 180^\circ$.

Click the image to the right to explore supplementary angles using Desmos Geometry.

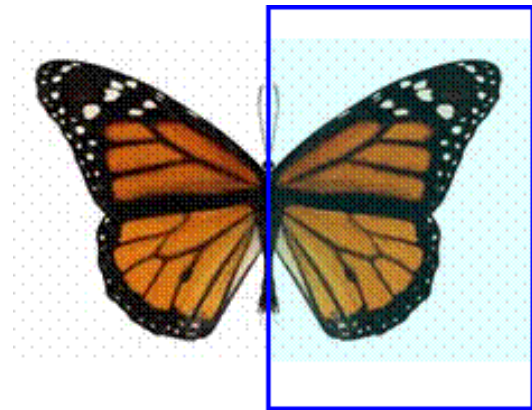


S

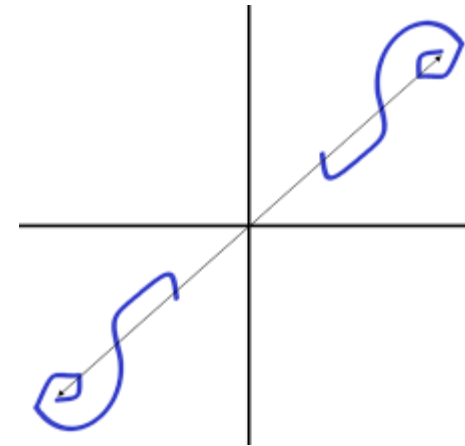
Symmetry. A line of symmetry is a line that passes through an object or shape, creating two congruent halves that are mirror images of one another. A point of symmetry is a central point on a shape or object. Every point on opposite sides is the same distance from the central point.

Examples:

This butterfly has one line of symmetry, creating two congruent halves.



Corresponding points of each image are equidistant from the point of symmetry (the origin).



T

[Tangent](#)

[Tape Diagram](#)

[Term](#)

[Terminating Decimal](#)

[Third Quartile](#)

[Tile/Tiling](#)

[Transitivity Principle for Indirect Measurement](#)

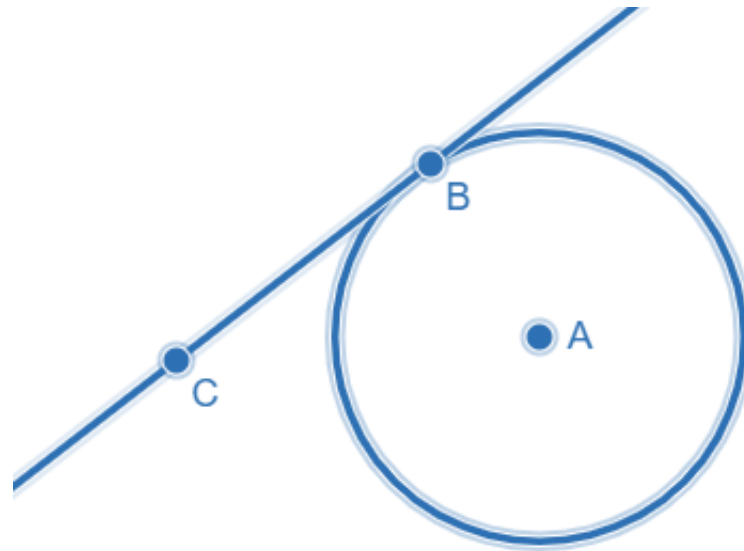
[Transversal](#)

[Trapezoid](#)

[Triangle](#)

T

Tangent. A line that touches a curve at one point. In the figure below, line CB is tangent to circle A at point B.



T

Tape Diagram. A visual representation of a situation or problem often used to make sense of and solve problems involving addition, subtraction, multiplication, and ratios.

Examples:

Andre spent \$86.21 on three gifts for his family. He spent \$21.09 on the first gift and \$33.45 on the second gift. How much money did he spend on the third gift?

Total = \$86.21		
1 st Gift	2 nd Gift	3 rd Gift
\$21.09	33.45	?

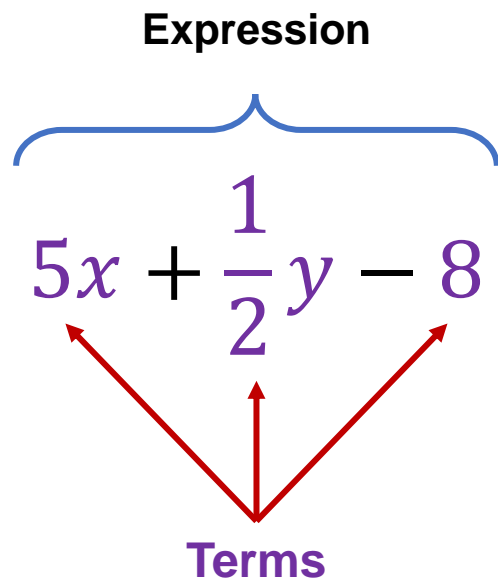
Jenise has found the perfect recipe for chocolate milk. It has 2 parts of chocolate to 3 parts of milk. Jenise wants to make a batch for a math party. She has 9 cups of milk. How many cups of chocolate does she need?



T

Term. A value on which mathematical operations take place in an expression. Also, a number in a sequence or series.

Examples:



Number Sequence:

7, 14, 21, 28, 35, 42, 49, 56, ...

14 is the second term
in this sequence.

T

Terminating Decimal. A decimal with a finite number of digits. Also, if a decimal's repeating digit is zero, the decimal is called a terminating decimal. Examples of terminating decimals can be seen below. See *also*: [Decimal](#), [Repeating Decimal](#)

Examples:

$$\frac{1}{4} = 0.25$$

$$\frac{5}{8} = 0.875$$

$$\frac{1}{2} = 0.5$$

$$\frac{52}{10} = 5.2$$

$$\frac{3256}{10} = 32.56$$

$$\frac{3}{5} = 0.6$$

$$\frac{75}{100} = 0.75$$

$$\frac{73}{1000} = 0.073$$

T

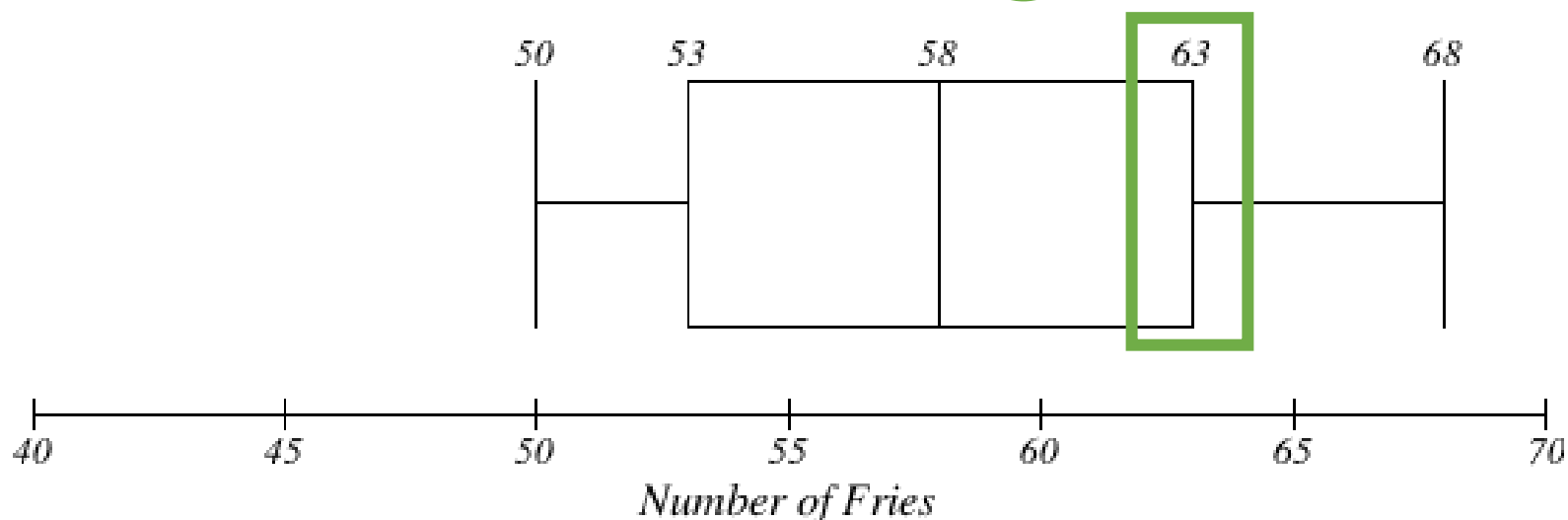
Third Quartile. For a data set with median M , the third quartile is the median of the data values between M and the Upper Extreme; also called *Upper Quartile*. See also: [Median](#), [First quartile](#), [Interquartile Range](#), [Quartile](#).

Example:

For the data set of the number of French fries in a small order of McDonalds fries, the third quartile is 63.

Number of French fries in a Small order of McDonalds Fries

50, 53, 53, 54, 56, 58, 59, 61, 63, 65, 68

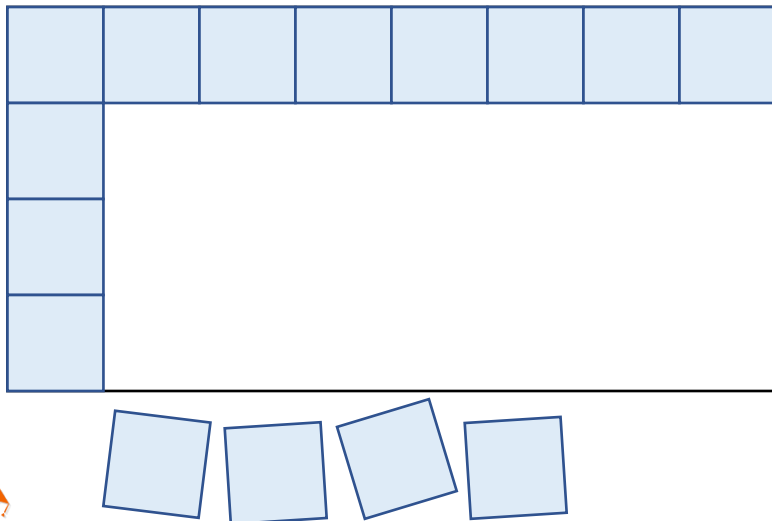


T

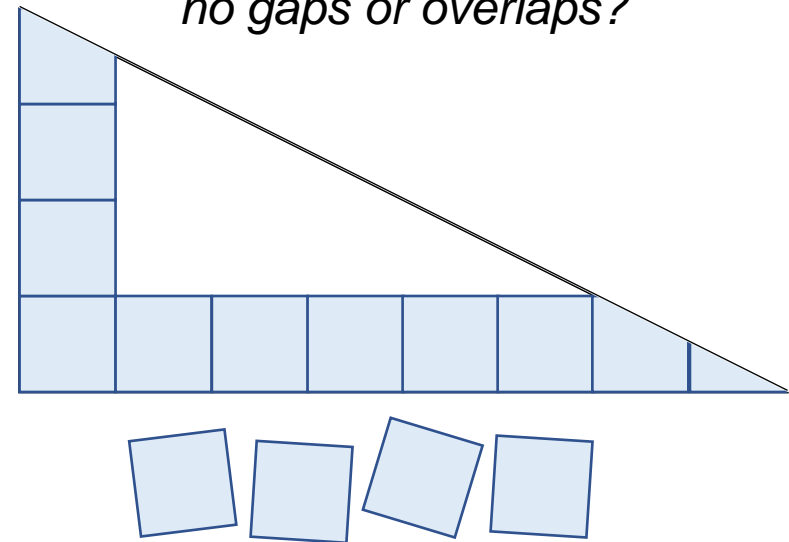
Tiling. Covering a space (area) by fitting individual tiles (color tiles, pattern blocks, or other plane figures) together with no gaps or overlaps.

Examples:

How many blue tiles do I need to completely cover the area of this rectangular space with no gaps or overlaps?



How many blue tiles do I need to completely cover the area of this triangular space with no gaps or overlaps?



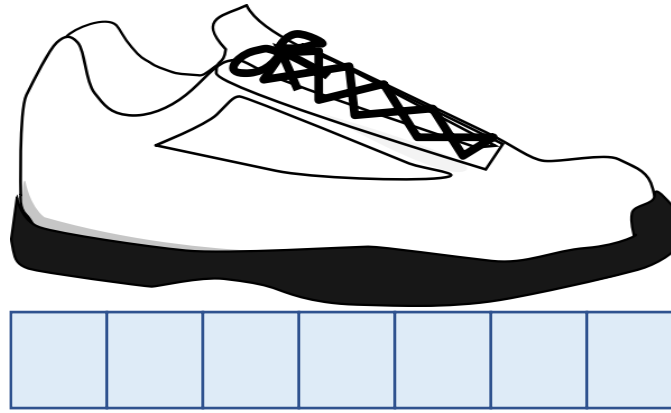
T

Transitivity Principle for Indirect Measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

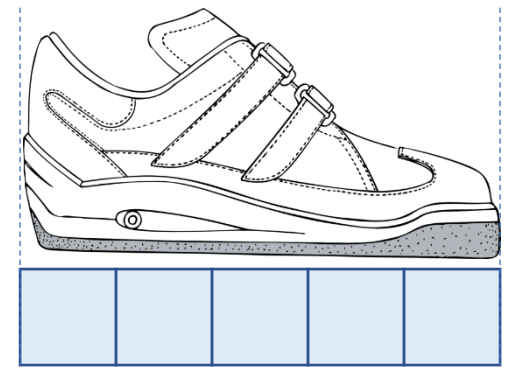
**Shoe A is bigger than shoe B.
Shoe B is bigger than shoe C.
So, shoe A is bigger than shoe C.**



Shoe A is 8 units long.



Shoe B is 7 units long.

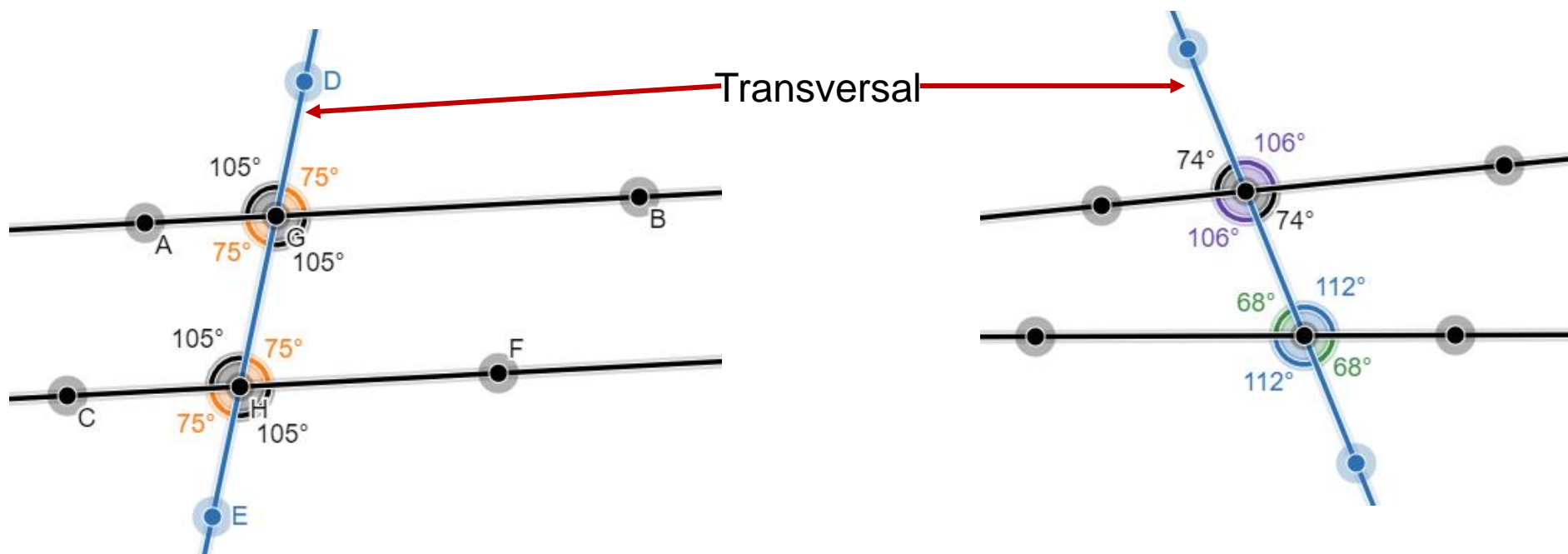


Shoe C is 5 units long.

T

Transversal. A line that intersects two or more lines.

Examples:

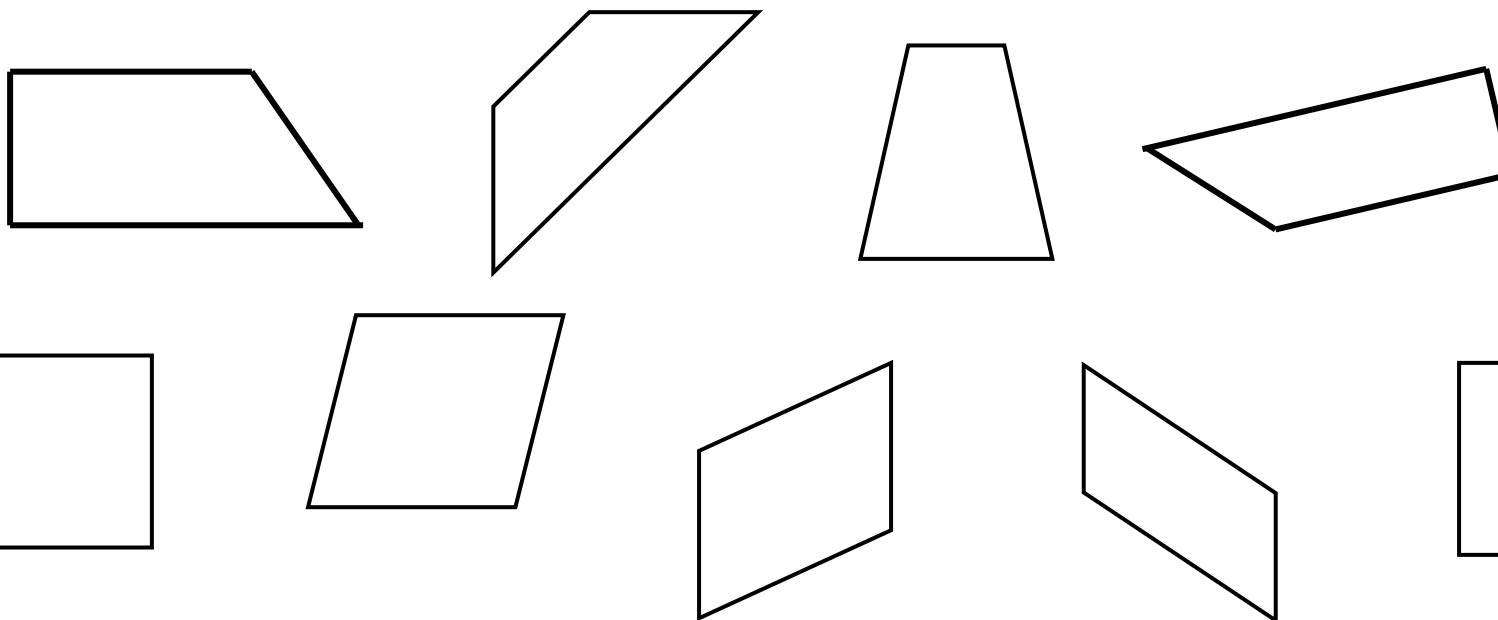


T

Trapezoid. A quadrilateral with at least one pair of parallel sides (*Georgia uses the inclusive definition of trapezoid*). See also: [Polygon](#)

Examples:

Click on a trapezoid below to explore this definition further.



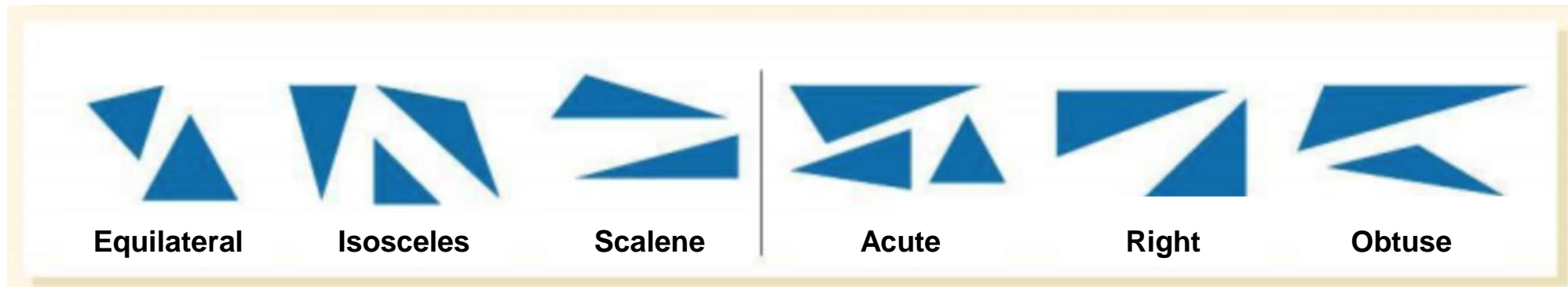
T

Triangle. A plane figure with three straight sides and three angles. A polygon with three straight sides. See also: [Polygon](#)

Examples:

Triangles Classified by Side Lengths

Triangles Classified by Angles



U

[Uniform Probability Model](#)

[Unit Fraction](#)

[Unitizing](#)

[Upper Extreme](#)

U

Uniform Probability Model. A probability model which assigns equal probability to all outcomes. See *also:* [Probability Model](#).

Below are a few examples of common uniform probability models:

Tossing a fair coin



All possible outcomes

Each outcome has an equal probability

$$\frac{1}{2}$$

Rolling a six-sided die



$$\frac{1}{6}$$

U

Unit Fraction. A fraction where the numerator is one and the denominator is a positive integer. See *also:* [Numerator](#), [Denominator](#).

Example:

$\frac{1}{5}$ of the lollipops below are red.



U

Unitizing. The counting and quantity principle that refers to the understanding that you can count a large group of items by counting smaller, equal groups of items from within the large group. *See also:*

[Place Value](#)

In our base-ten system, we unitize:



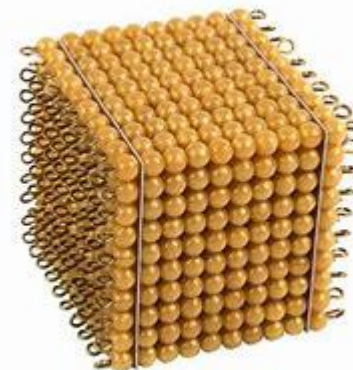
One bead.



Ten beads are grouped together and can be thought of as one 10.



One-hundred beads are grouped together and can be thought of as one 100.

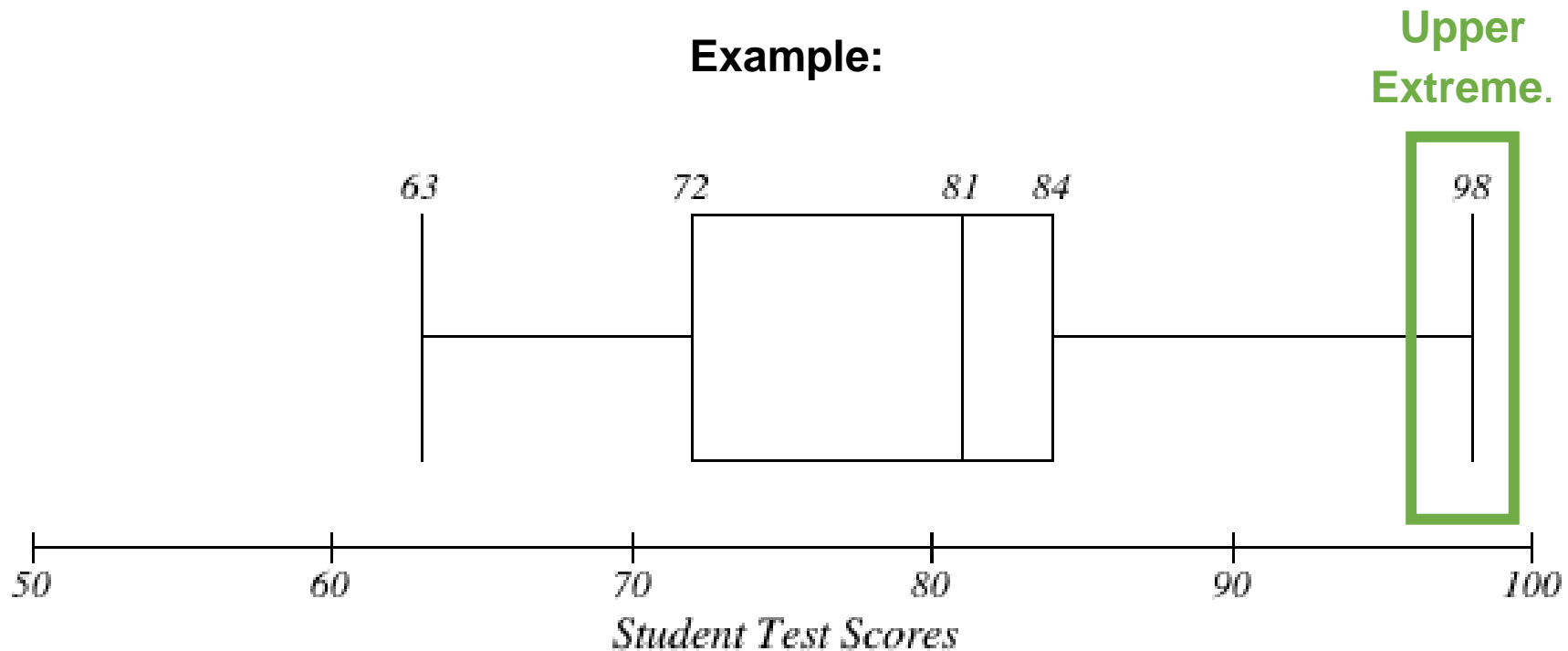


One-thousand beads are grouped together and can be thought of as one 1,000.

U

Upper Extreme. The largest value in the data set; also called the maximum. See *also*: [Lower Extreme](#).

Example:



V

[Variable](#)

[Variability](#)

[Variance](#)

[Vector](#)

[Vertex](#)

[Visual Fraction Model](#)

[Volume](#)

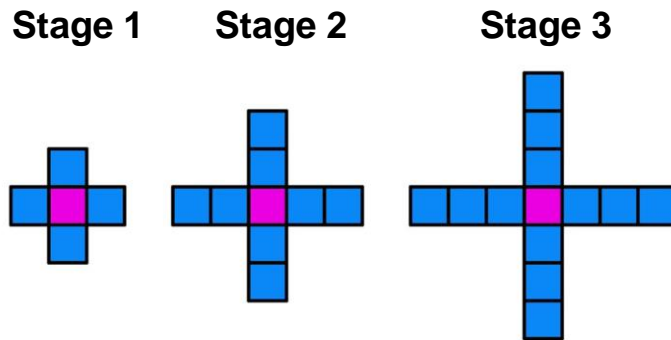


[Back to Start](#)

V

Variable. A symbol for a value that is unknown. It is usually represented with a letter such as x or y .

Example:



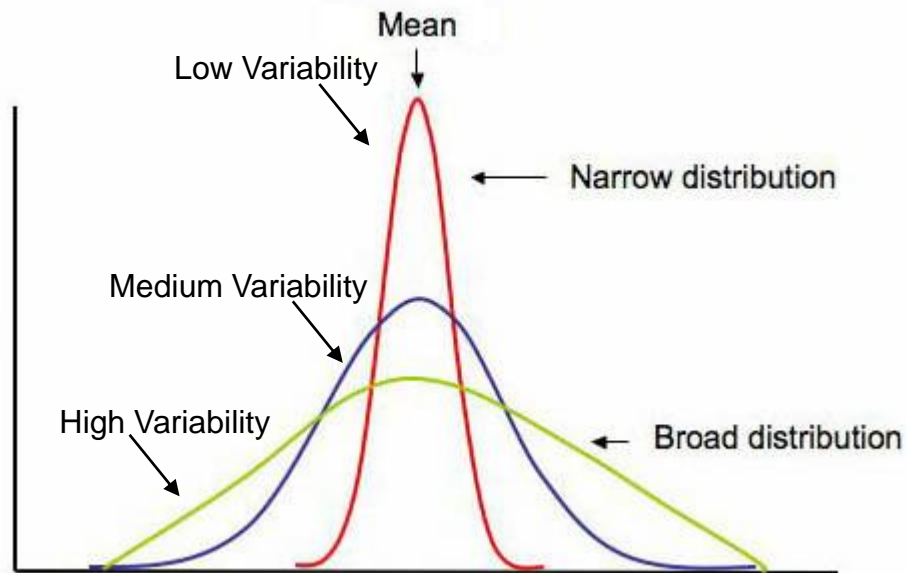
The variable n , in this table, represents any unknown stage number

Stage	Stays the Same (Constant)	Changes (Variable)	Total
1	1	4	5
2	1	8	9
3	1	12	13
4	1	16	17
5	1	20	21
...
17	1	$17 \times 4 = 68$	69
29	1	$29 \times 4 = 116$	117
n	1	$n \times 4 = 4n$	$4n + 1$

The variable n is used here, with the patterns discovered, to create an algebraic expression that can be used to determine the total squares needed for any stage.

V

Variability. How spread out or closely clustered a set of data is. Some common measures of variability are [Range](#), [Interquartile Range](#), [Variance](#), and [Standard Deviation](#).

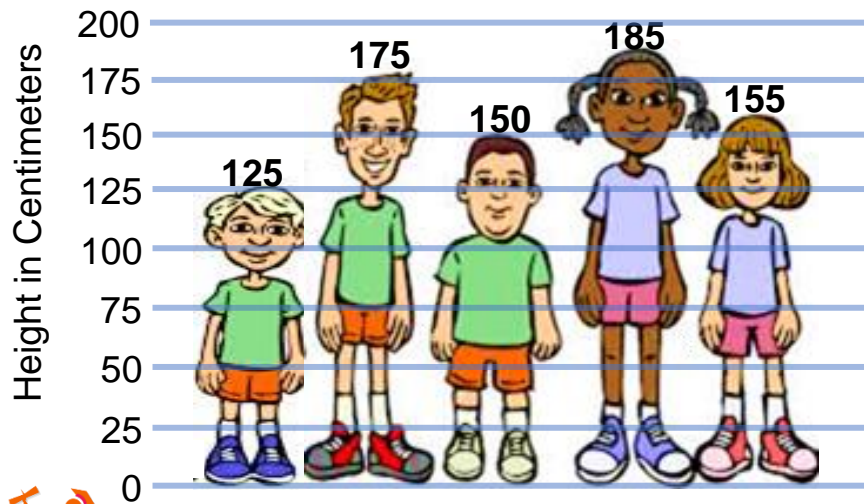


V

Variance. A measurement of the spread between numbers in a data set. The average of the squared differences from the mean. See also: [Mean](#), [Standard Deviation](#).

Example:

Five friends measured their height to the nearest cm as shown below.



① Find the mean of the friends' heights.

$$\begin{aligned}\text{Mean} &= \frac{125+150+155+175+185}{5} \\ &= 158 \text{ cm}\end{aligned}$$

③ Find the variance: Take each difference and square it, then find the average:

$$\frac{33^2+8^2+3^2+17^2+27^2}{5} = 436$$

② Find the distance from the mean for each person's height:

$$\begin{aligned}158-125 &= 33 \text{ cm} \\ 158-150 &= 8 \text{ cm} \\ 158-155 &= 3 \text{ cm} \\ 175-158 &= 17 \text{ cm} \\ 185-158 &= 27 \text{ cm}\end{aligned}$$

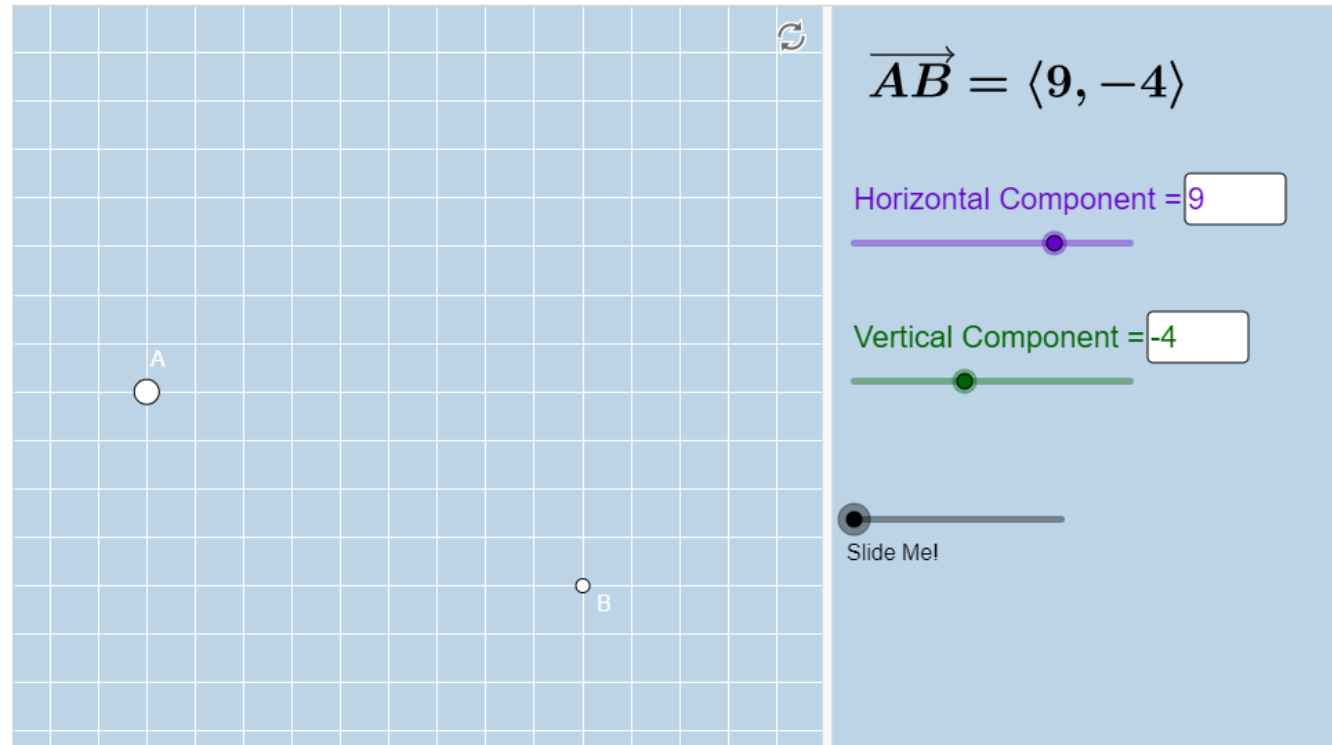
④ Looking at this small data set, the data is not too spread out. The variance and standard deviation can provide more information about how spread out the data are.

V

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Example:

Click the image to try
an interactive vector
activity using
GeoGebra.

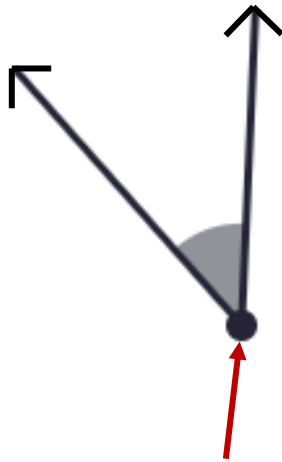


[Back to Start](#)

V

Vertex. A common endpoint of two or more rays or line segments. A vertex in a polygon is where two sides meet. A vertex in a solid figure is where three or more edges meet. *Plural:* Vertices.

Examples:

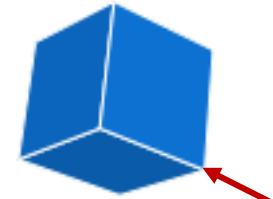


An angle has one vertex.



One of the pentagon's 5 vertices.

A pentagon has 5 corners. Each one is a vertex.



One of the cube's 8 vertices.

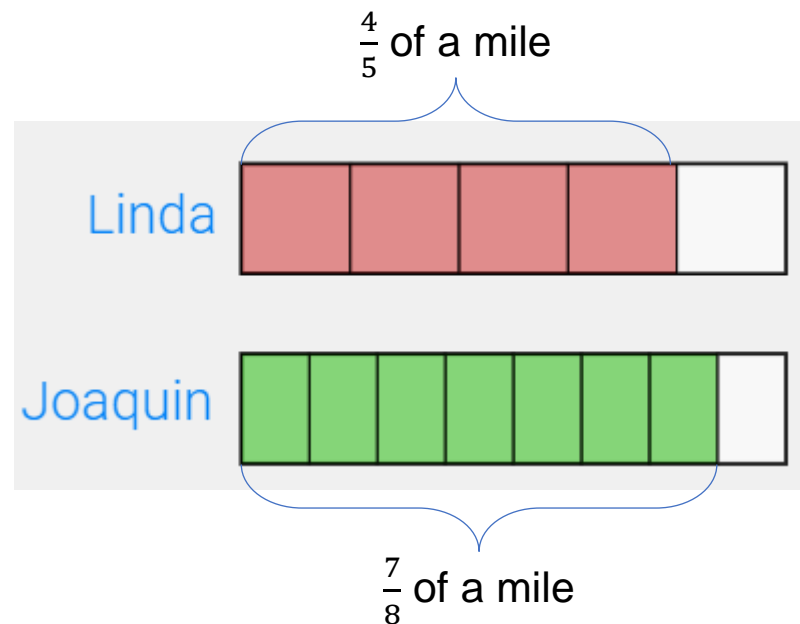
A cube has 8 corners. Each one is a vertex.

V

Visual Fraction Model. A visual representation of a fraction or fractions (i.e., a tape diagram, number line diagram, area model, etc.) often used to make sense of and solve problems involving fractions.

Example:

Linda ran $\frac{4}{5}$ of a mile. Joaquin ran $\frac{7}{8}$ of a mile. Who ran farther?

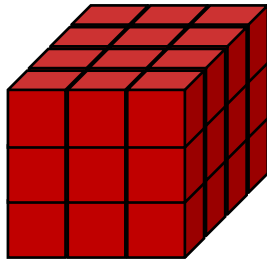


Joaquin ran farther because $\frac{1}{5}$ is a greater distance from the whole mile than $\frac{1}{8}$ is.

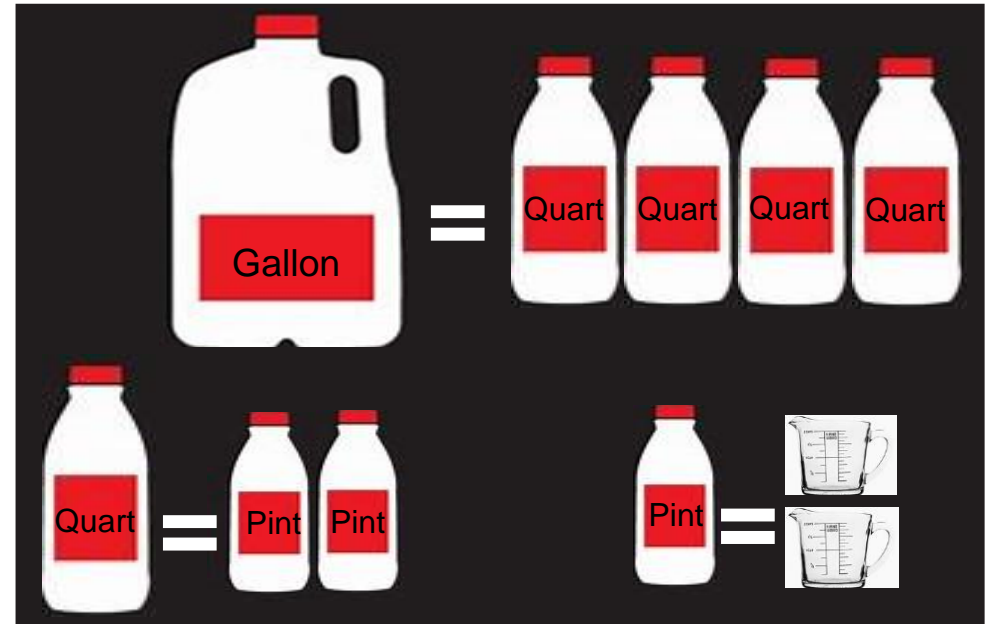
V

Volume. A unit of measure describing how much three-dimensional space a substance occupies or the capacity of a container, provided in cubic units.

Example:



This rectangular prism has 36 cubes. Its volume is 36 cubic units.

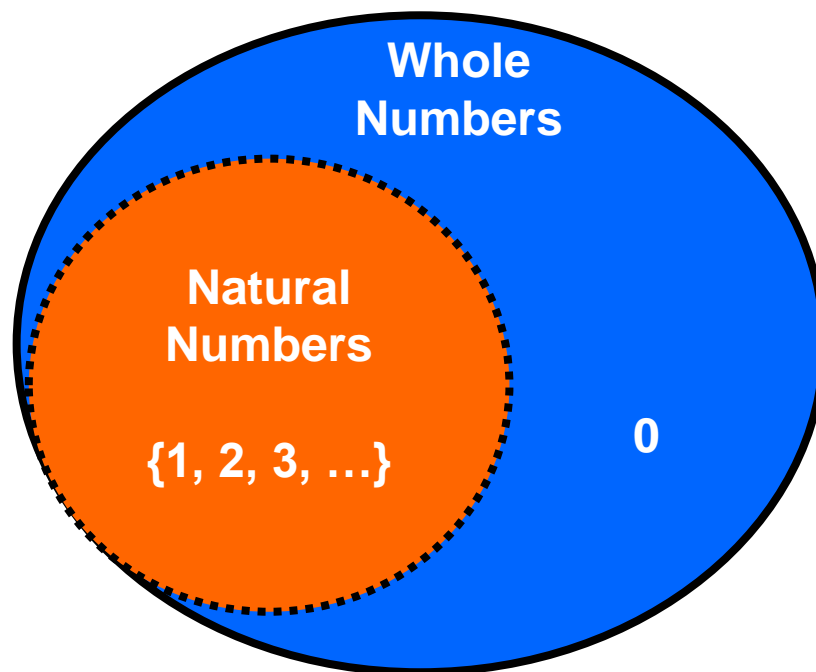


W

Whole Numbers

W

Whole Numbers. The numbers 0, 1, 2, 3, The whole numbers include the Natural Numbers or Counting Numbers and zero.



X

x-axis

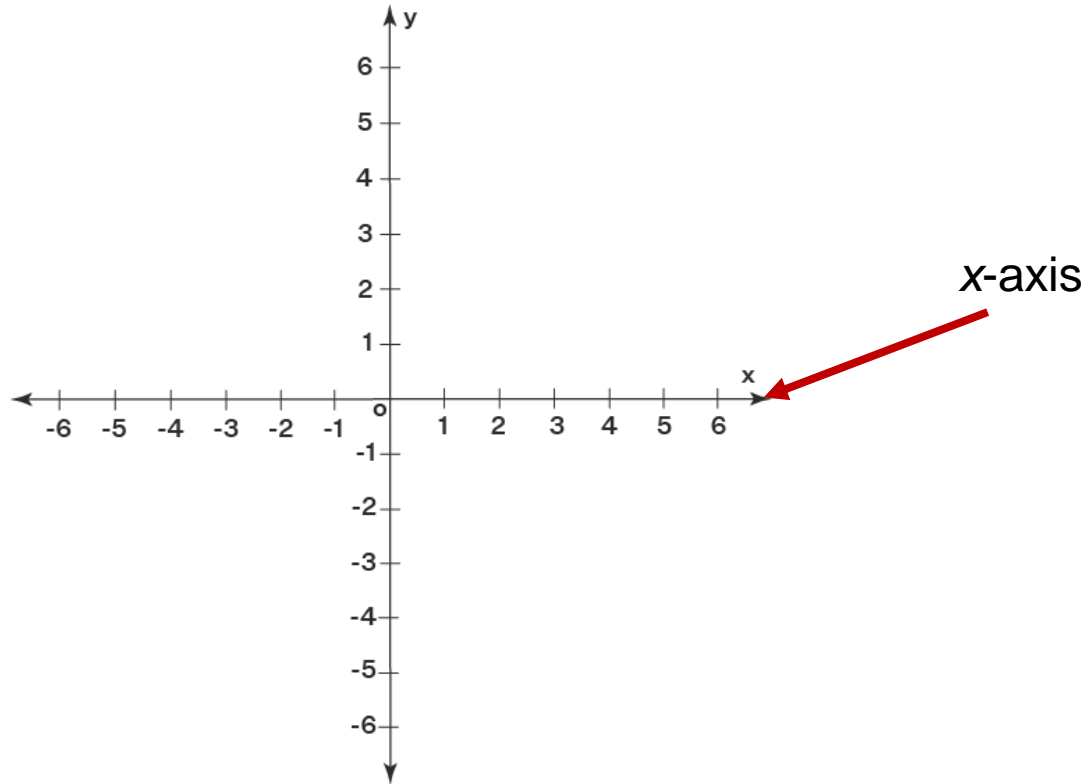
x-coordinate

x-intercept

X

x-axis. The horizontal number line which, along with the vertical number line (y-axis), creates the coordinate plane. Together, these two axes create a system for identifying the precise locations of points on the coordinate plane. The x-axis is often referenced as the independent axis.

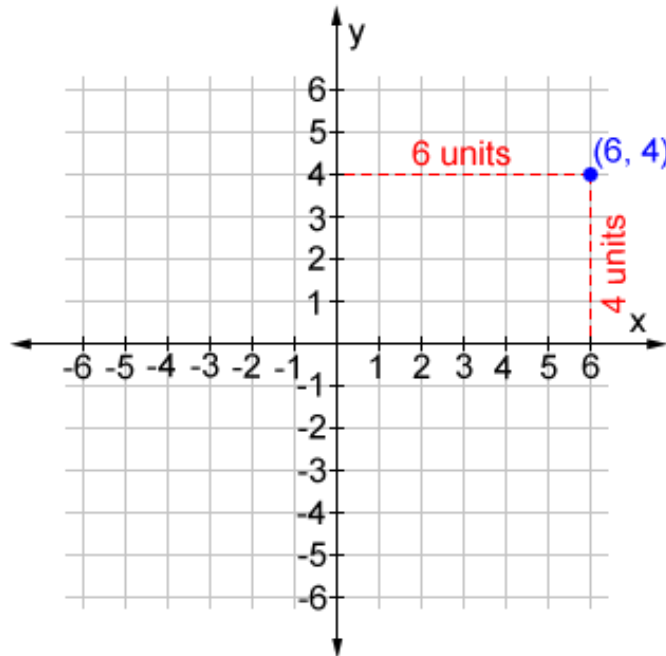
See also: [y-axis](#), [Coordinate](#).



X

x-coordinate. The horizontal distance of a given point from the y-axis. In an ordered pair of coordinates, the x-coordinate is the first number.

Example:

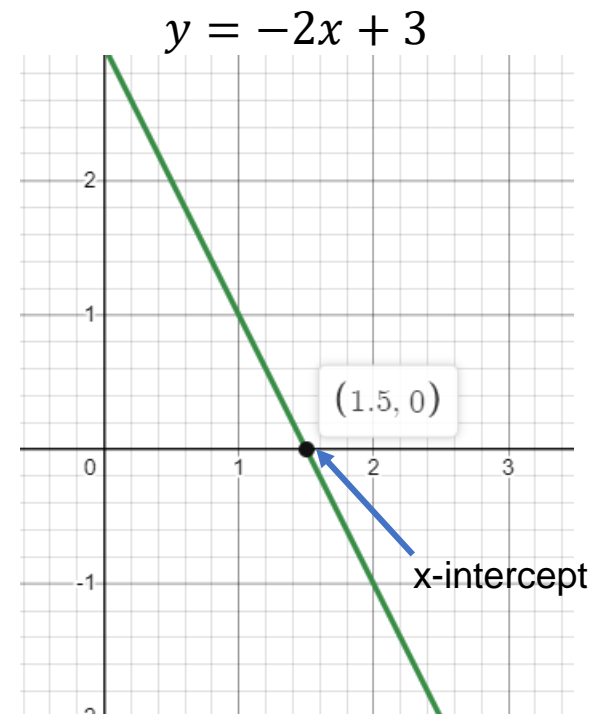
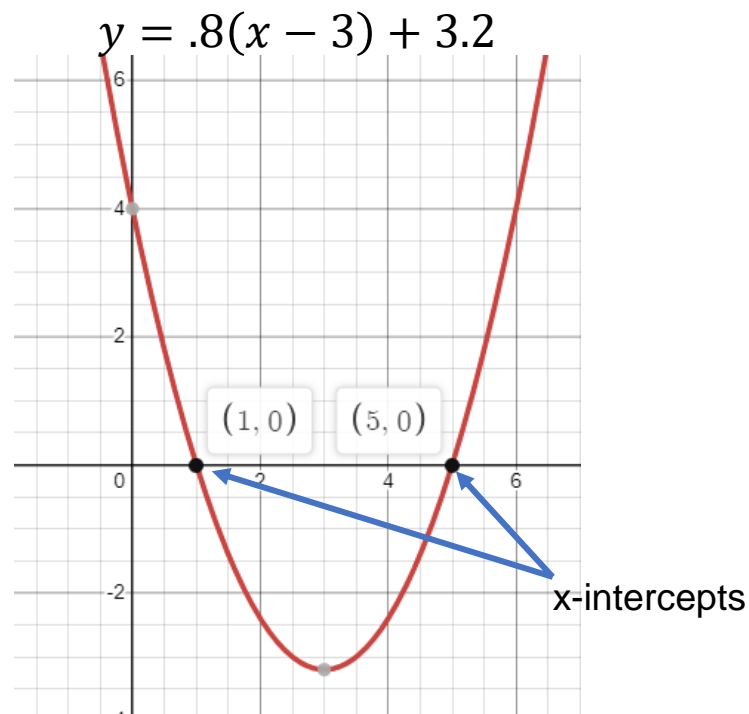


The blue point in the graph above is located by the ordered pair (6, 4). **6 is the x-coordinate** and 4 is the y-coordinate.

X

x-intercept. The point or points at which the graph of an equation crosses the x-axis. For a point to cross the x-axis it must have a y-value of 0. As a result, an x-intercept can always be represented by $(x, 0)$. The x-value indicates where the graph crosses the x-axis.

Examples:



Y

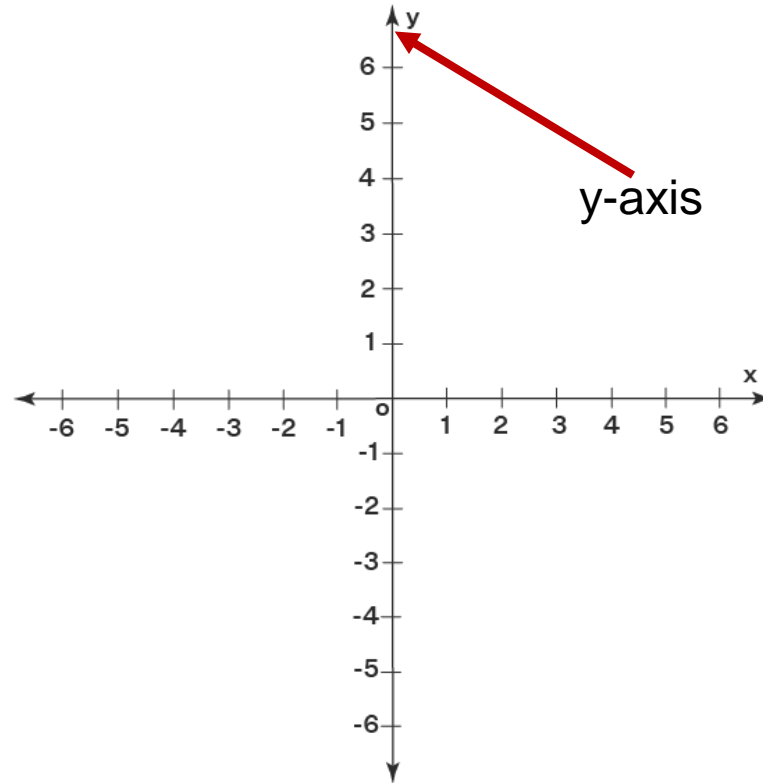
[y-axis](#)

[y-coordinate](#)

[y-intercept](#)

Y

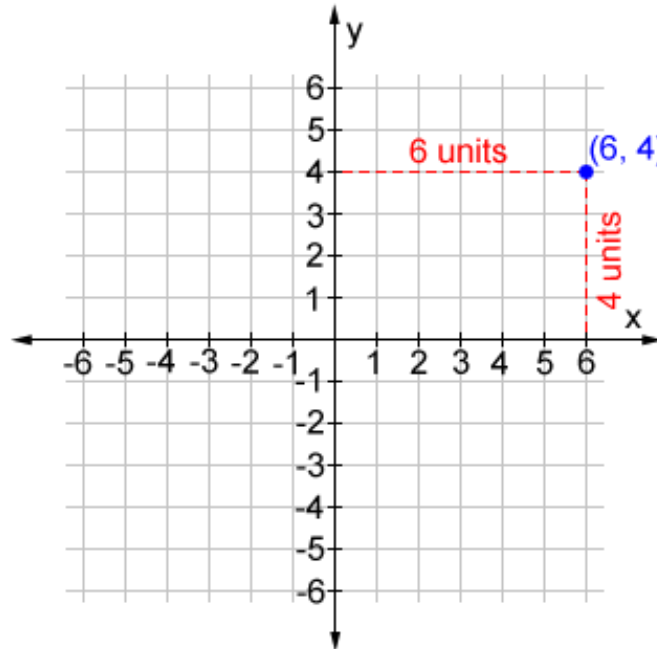
y-axis. The vertical number line which, along with the horizontal number line (x-axis), create the coordinate plane. Together, these two axes create a system for identifying the precise locations of points on the coordinate plane. The y-axis is often referenced as the dependent axis.



Y

y-coordinate. The vertical distance of a given point from the x-axis. In an ordered pair of coordinates, the y-coordinate is the second number. See *also*: [x-axis](#), [Coordinate](#).

Example:

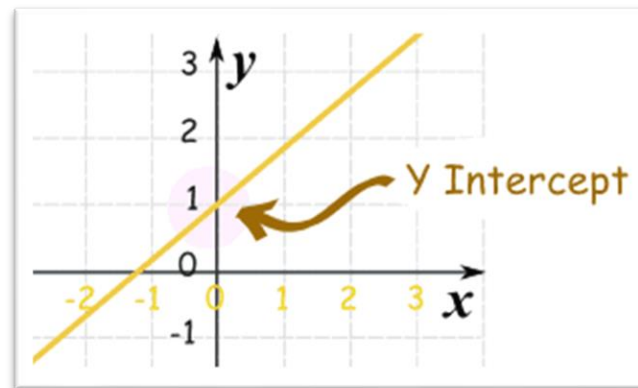


The blue point in the graph above is located by the ordered pair, (6, 4). 6 is the x-coordinate and **4 is the y-coordinate.**

Y

y-intercept. The point at which the graph of an equation crosses the y -axis. For a point to cross the y -axis it must have an x value of 0. As a result, a y -intercept can always be represented by $(0, y)$. The y value indicates where the graph crosses the y -axis.

Example:



Click the image to try an interactive area model for y-intercept the Desmos Graphing Calculator.

In the above diagram the line crosses the y -axis at 1. So, the y -intercept is equal to 1.

Z

[z-axis](#)

[z-coordinate](#)

[z-intercept](#)

[z-score](#)

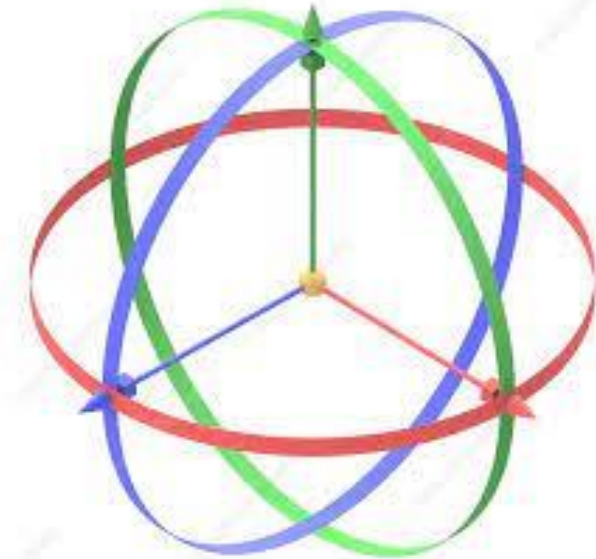
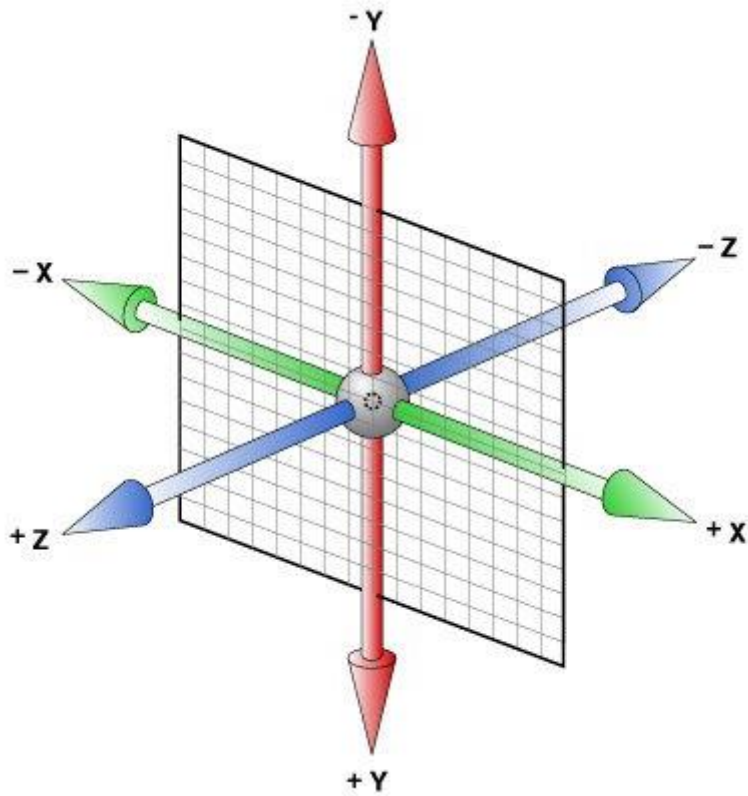
[Zero](#)

[Zero Angle](#)

[Zero Pair](#)

Z

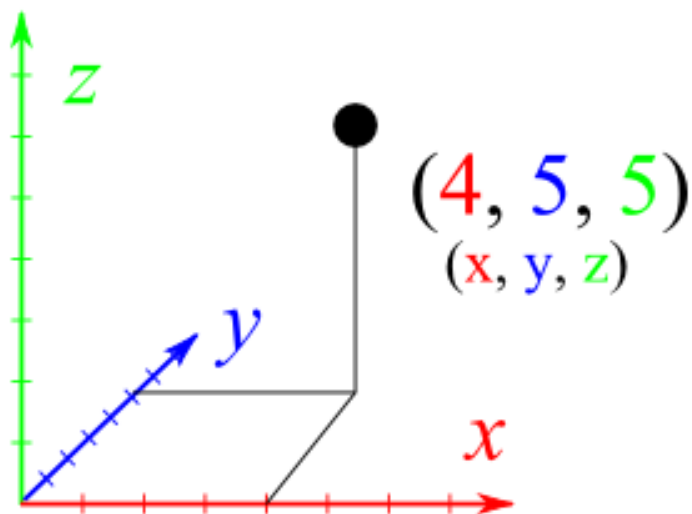
z-axis a reference axis of a three-dimensional Cartesian coordinate system; the axis along which values of z are measured and at which both x and y equal zero.



Z

z-coordinate. The perpendicular distance of a given point from the (x, y) ordered pair coordinates of the point. In an ordered triple of coordinates, the z-coordinate is the third number.

Example:

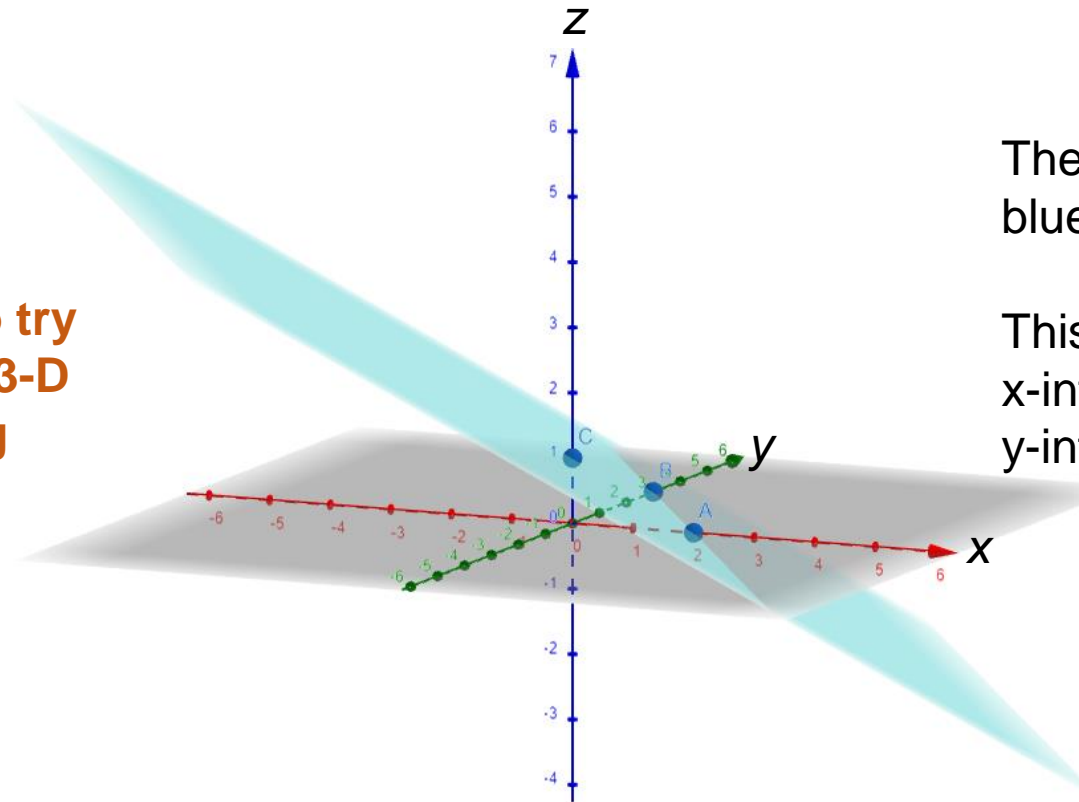


Z

z-intercept. The point at which the graph of an equation crosses the z-axis. For a point to cross the z-axis it must have an x value of 0 and a y value of 0. As a result, a z-intercept can always be represented by $(0, 0, z)$. The z value indicates where the graph crosses the z-axis.

Example:

Click the image to try some interactive 3-D graphing using GeoGebra.



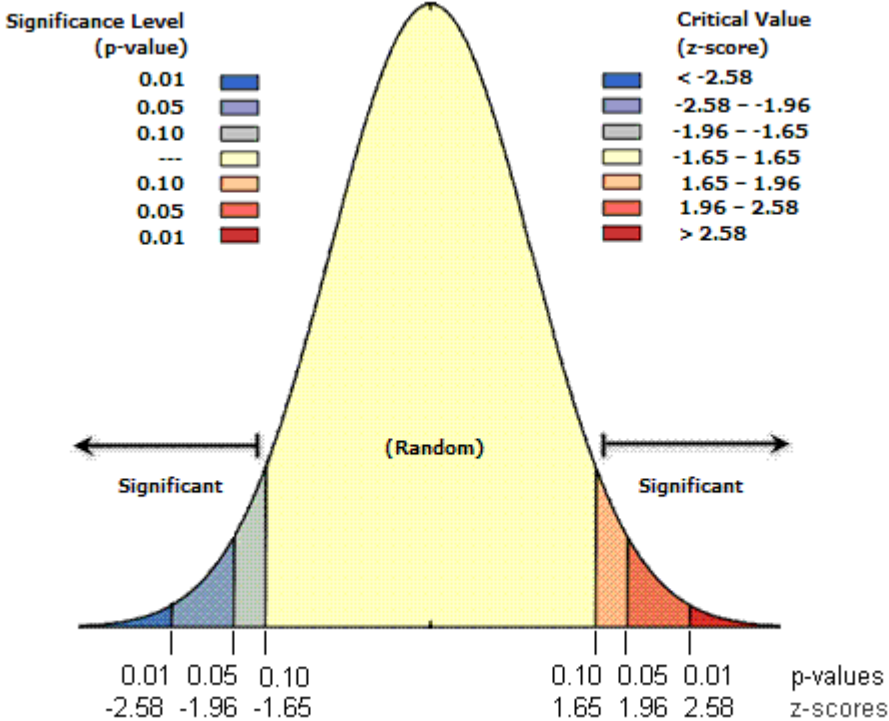
The z-intercept of this light blue plane is $(0, 0, 1)$.

This plane also has an x-intercept of $(2, 0, 0)$ and a y-intercept of $(0, 3, 0)$.

Z

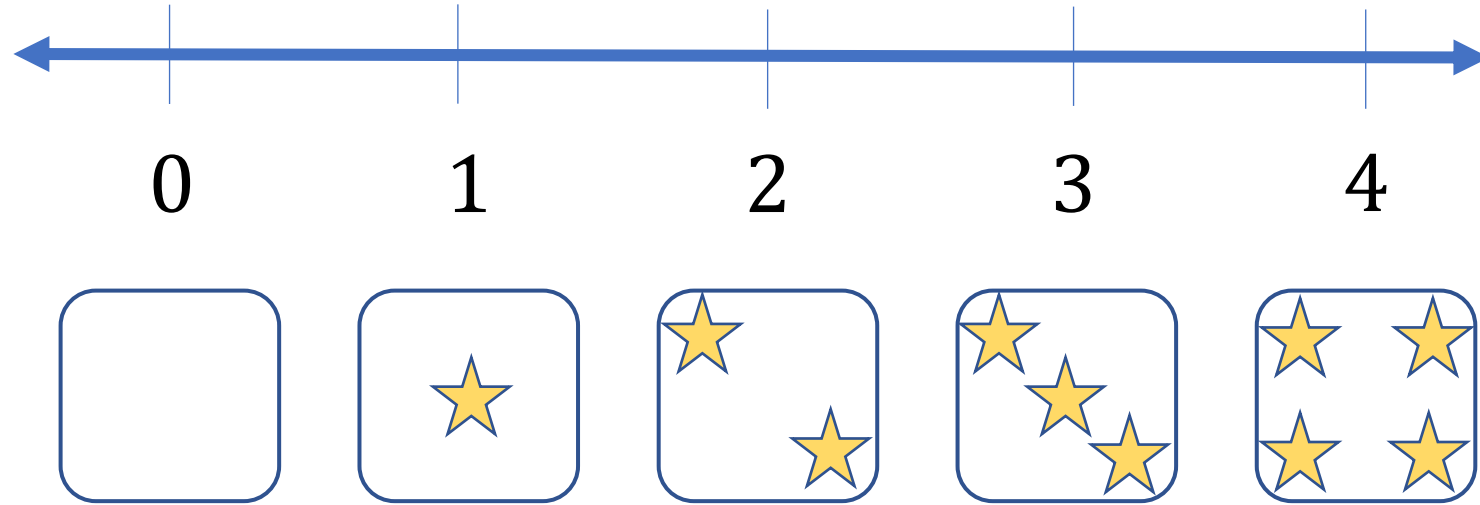
z-score. a measure of standard deviations from the mean. If a z-score is 0, it indicates that the data point's score is equal to the mean score. A z-score of 1.0 would indicate a value that is one standard deviation from the mean.

Click the image to try an interactive area model for z-scores using GeoGebra.



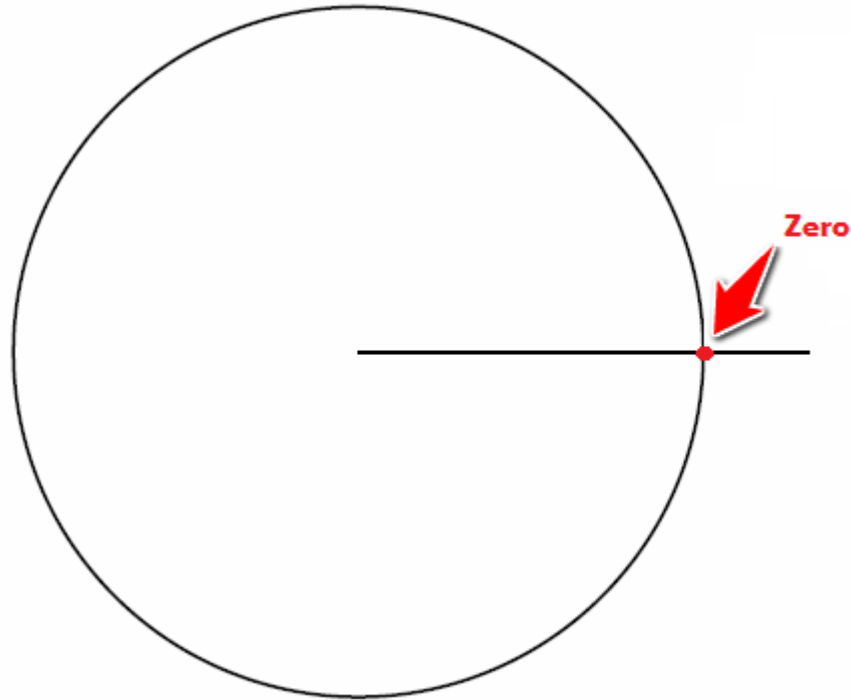
Z

Zero. A quantity that represents an empty set. The mathematical symbol “0” is used to denote the absence of all magnitude or quantity.



Z

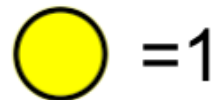
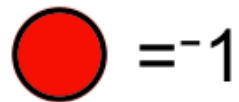
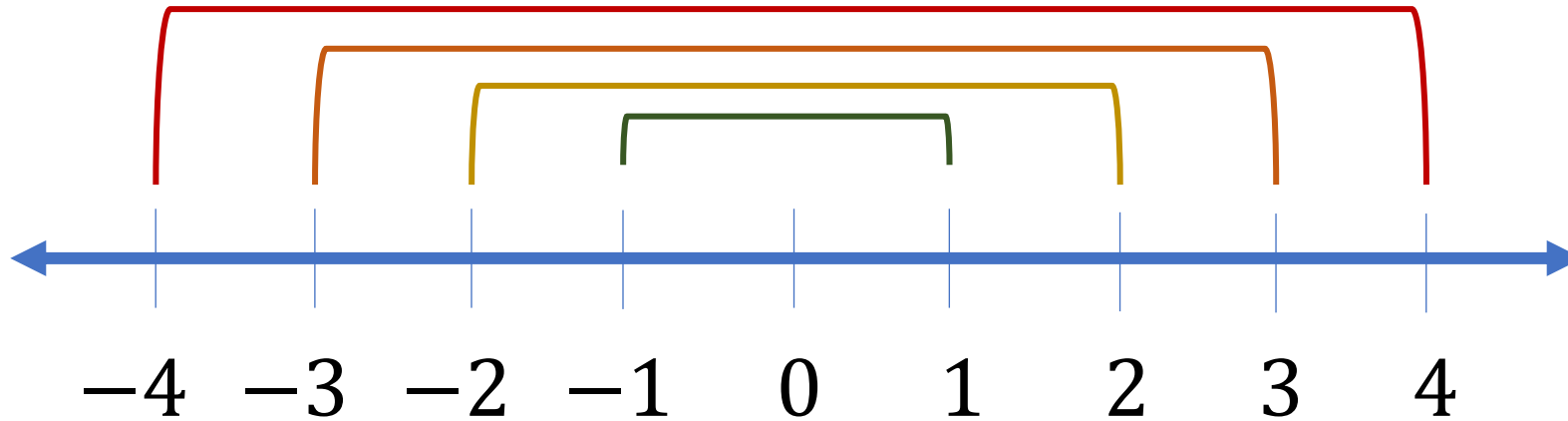
Zero Angle. An angle with a measure of zero. The measure can be represented in three different forms mathematically: zero degrees: 0° ; zero radians: 0 ; zero gradians: 0^g . See *also*: [Angle](#).



Z

Zero Pair. A pair of numbers whose sum is zero. In the number line image below, numbers connected with a colored bracket are zero pairs. Visually, zero pairs can also be represented with two-color counters as seen below the number line.

Examples:



$=$ zero pair

Problem Types for Addition, Subtraction, Multiplication, and Division

Click the image to view the full-size document.

Mathematics Problem Types		
Joining Problems (Action)		
<p>(Result Unknown)</p> <p>Ann had 6 pencils. Juan gave her 8 more pencils. How many pencils does Ann have all together?</p> <p>$6 + 8 = \square$</p>	<p>(Change Unknown)</p> <p>Ann had 8 pencils. Juan gave her some more pencils. Now she has 14 pencils. How many pencils did Juan give Ann?</p> <p>$8 + \square = 14$</p>	<p>(Start Unknown)</p> <p>Ann had some pencils. Juan gave her 8 more pencils. Now she has 14 pencils. How many pencils did Ann have to start with?</p> <p>$\square + 8 = 14$</p>
Separating Problems (Action)		
<p>(Result Unknown)</p> <p>Ann had 14 goldfish. He gave 8 goldfish to his friend, Juan. How many goldfish does Ann have now?</p> <p>$14 - 8 = \square$</p>	<p>(Change Unknown)</p> <p>Ann has 14 goldfish. He gave some goldfish to Juan. Now he has 8 goldfish. How many goldfish did he give to Juan?</p> <p>$14 - \square = 8$</p>	<p>(Start Unknown)</p> <p>Ann had some goldfish. He gave 8 goldfish to Juan. Now he has 6 goldfish. How many goldfish did Ann have to start with?</p> <p>$\square - 8 = 6$</p>
Part-Part-Whole Problems (No Action)		
<p>(Whole Unknown)</p> <p>Ann has 7 blue cap erasers and 8 pink cap erasers. How many cap erasers does Ann have?</p> <p>$7 + 8 = \square$</p>	<p>(Part Unknown)</p> <p>Ann has 15 cap erasers. 8 are pink and the rest are blue. How many blue cap erasers does she have?</p> <p>$15 - 8 = \square$ OR $8 + \square = 15$</p>	<p>(Both Parts Unknown)</p> <p>Ann has 15 cap erasers. Some are pink and some are blue. How many could be pink and how many could be blue?</p> <p>$\square + \square = 15$</p>
Compare Problems (No Action)		
<p>(Difference Unknown)</p> <p>Ann has 12 marbles. Juan has 7 marbles. How many more marbles does Ann have than Juan?</p> <p>$12 - 7 = \square$ OR $7 + \square = 12$</p>	<p>(Quantity Unknown)</p> <p>Ann has 5 marbles. Juan has 7 more than Ann. How many marbles does Juan have?</p> <p>$5 + 7 = \square$</p>	<p>(Referent Unknown)</p> <p>Juan has 12 marbles. He has 5 more than Ann. How many marbles does Ann have?</p> <p>$12 - 5 = \square$ OR $5 + \square = 12$</p>
Groups		
<p>(Multiplication)</p> <p>Ann has 3 packages of gummies. Each package contains 8 gummies. How many gummies does Ann have?</p> <p>$3 \times 8 = \square$</p>	<p>(Measurement Division)</p> <p>Ann has 24 gummies. There are 8 gummies in each bag. How many bags of gummies does she have?</p> <p>$24 \div 8 = \square$</p>	<p>(Partitive Division)</p> <p>Ann has 3 bags of gummies with the same number of gummies in each bag. Altogether, she has 24 gummies. How many gummies are in each bag?</p> <p>$24 \div 3 = \square$</p>

Properties of Operations

Associative Property of Addition	$(a + b) + c = a + (b + c)$
Commutative Property of Addition	$a + b = b + a$
Additive Identity Property of 0	$a + 0 = 0 + a = a$
Existence of Additive Inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$.
Associative Property of Multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative Property of Multiplication	$a \times b = b \times a$
Multiplicative Identity Property of 1	$a \times 1 = 1 \times a = a$
Existence of Multiplicative Inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$.
Distributive Property	$a \times (b + c) = a \times b + a \times c$

Properties of Equality

Reflexive Property of Equality	$a = a$
Symmetric Property of Equality	If $a = b$, then $b = a$.
Transitive Property of Equality	If $a = b$ and $b = c$, then $a = c$.
Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $a \times c = b \times c$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution Property of Equality	If $a = b$, then b may be substituted for a in any expression containing a .

Properties of Inequality

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.

If $a > b$ and $b > c$ then $a > c$.

If $a > b$, then $b < a$.

If $a > b$, then $-a < -b$.

If $a > b$, then $a \pm c > b \pm c$.

If $a > b$ and $c > 0$, then $a \times c > b \times c$.

If $a > b$ and $c < 0$, then $a \times c < b \times c$.

If $a > b$ and $c > 0$, then $a \div c > b \div c$.

If $a > b$ and $c < 0$, then $a \div c < b \div c$.